

# Comparing Cooling of Hot Mix and Warm Mix Asphalts

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Warm mix asphalt is increasingly being used in highway paving in the United States. Apart from environmental benefits, warm mix asphalt is being considered for use in areas where its material characteristics may help to overcome some of the challenges faced when using hot mix asphalt. Its potential for long haul and late season projects hinges on the rate of cooling during field compaction. In this paper we report our investigation into whether warm mix asphalt cools at a slower rate than hot mix asphalt. Our analysis of field observations from two paving projects indicated some types of warm mix asphalt would cool at a slower rate than hot mix asphalt under a given set of conditions. This suggests relatively slow cooling of warm mix asphalt may allow for sufficient compaction time required for achieving proper density under adverse conditions. The findings presented here can be informative in assessing the potential of warm mix asphalt for use in long haul, nighttime, and late season paving.

**Keywords:** Asphalt pavements; Compaction; Density; Thermal diffusion.

## Introduction

One of the determinants of long-term performance of asphalt pavements is density or in-place air voids of asphalt mixes (Bell et al. 1984). Compacting asphalt mixes to a low air voids level helps to minimize fatigue cracking, thermal cracking, and moisture susceptibility of asphalt pavements in service. As such, density has long been used as a construction quality characteristic of asphalt pavements to determine conformance with performance requirements (Von Quintus et al. 2009). The ability to compact an asphalt mix to proper density is influenced by many factors, such as the grade and content of asphalt binder, and moisture content and proportion of aggregates. However, temperature of an asphalt mix during paving is also an important factor in achieving proper density (Chadborn et al. 1998, Hughes 1989, Schmitt et al. 2009). The importance of asphalt mix temperature is well known to the paving industry and is reflected in their best practices. For example, the breakdown roller strives to keep up the pace of mix laydown while making a sufficient number of passes although success of such efforts may only be determined (and rewarded) upon density measurements of core samples.

Compactive efforts required to achieve adequate density on a given mix depend on mix characteristics (e.g., stiffness) but also mix temperature during paving. As such, it is useful in planning paving operations to know the time that the mix will take to cool to cessation temperature, a temperature at which the mix may no longer increase in density despite compactive efforts. As a matter of fact, cooling time of hot mix asphalt (HMA) can be predicted using any one of the computational methods available, discussed later. However, application of these methods does not readily extend to relatively new, warm mix asphalt (WMA), which is being increasingly used in highway paving. One reason is that while the existing methods require knowledge of thermal properties of a mix, it is unknown whether or not WMA has similar thermal properties to HMA.

This paper compares cooling of HMA and WMA as recorded with an infrared camera on two paving projects in North Dakota. Its primary focus is to determine whether or not WMA cools more slowly than HMA under similar field conditions. The findings can be useful for the asphalt paving industry in evaluating the potential of WMA for use in long haul projects and late season or nighttime paving where relatively slow cooling of WMA may present sufficient time to compact properly.

## **Background and Literature Review**

Although there are many factors affecting compaction of asphalt pavement, temperature of an asphalt mix during paving is considered one of the most important factors in achieving proper density (Chadbourn et al. 1998, Hughes 1989, Schmitt et al. 2009). For a given compactive effort, the lower the temperature of the mix at the time of compaction, the lower the density obtained (McLeod 1967). More recently, Willoughby et al. (2001) indicated that when an asphalt mat varies in temperature by 25°F (14°C) or greater, the relatively cold mat area may not be compacted to the same density level as the surrounding area. When the mix temperature falls below what is referred to as cessation temperature, compaction may no longer be able to increase density (Dickson and Corlew 1970). Often cited as cessation temperature is “an average layer temperature of 175°F (80°C)” below which additional compaction is uneconomical or injurious to the pavement (Tegeler and Dempsey 1973).

Compaction time limited by adverse field conditions can pose a challenge during asphalt pavement construction. Although mix temperature can be increased at the plant, excessively high mix temperature can damage asphalt binder and cause the mix to be undesirably tender (APEC 2000). Despite the limited ability to increase mix temperature, the asphalt paving industry has been achieving specified density through the use of best practices. For example, keeping the rollers directly behind the paver helps to maximize the use of available compaction time (Scherocman 2006).

There have been many research efforts to predict available compaction time (Chadbourn et al. 1998; Chang et al. 2009; Corlew and Dickson 1968; Jordan and Thomas 1976; and Tegeler and Dempsey 1973). Except for Chang et al. (2009), researchers modeled asphalt mat cooling during paving as transient heat flow in which the temperature within the mat varies in both space and time. The governing equation of transient heat flow says that the time rate of temperature change at a point within the material, i.e., cooling rate, is proportional to the net heat flow through that point (Powers 2006). This proportionality is represented in the equation by the material's thermal diffusivity, which indicates how quickly the material carries heat away and is calculated from three terms: thermal conductivity, specific heat, and density. Thermal diffusivity of HMA ranges from  $0.37 \times 10^{-6}$  to  $1.44 \times 10^{-6} \text{ m}^2/\text{s}$ , as Chadbourn et al. (1998) estimated based on experimental determination of thermal conductivity of HMA.

The heat diffusion equation, coupled with appropriate initial and boundary conditions, can be solved using different numerical methods. The resulting solution is often presented in a cooling curve that plots average asphalt mat temperature against time. Then given as available compaction time for anticipated environmental (boundary) conditions is the time it takes the asphalt mat upon laydown (initial condition) to cool to cessation temperature. It should be noted however that application of the existing numerical

methods in estimating available compaction time for WMA requires knowledge of its thermal diffusivity value. Alternatively, thermal diffusivity of WMA would have to be assumed to be similar to that of HMA, which contrasts with the general belief that the relatively low viscosity of WMA reduces temperature dependency of compaction. With a chemical additive, wax or water introduced into the production process, WMA can achieve low viscosity relative to the production temperature that typically lies in the range of 220 to 275°F (104 to 135°C), which is as much as 100°F (56°C) lower than HMA production temperatures. For details of environmental and other potential benefits of WMA, the reader is referred to Kristjansdottir et al. (2007). However, WMA has been not sufficiently tested in adverse field conditions, and it remains uncertain whether or not WMA cools at a sufficiently low rate to allow for sufficient compaction time.

In summary, available compaction time is important information in achieving proper density for desirable long-term performance of asphalt pavements. Although existing numerical methods can be used to predict available compaction time for HMA, extending their application to increasingly used WMA requires as one input its thermal diffusivity, which may or may not differ from that of HMA. The objective of the research presented here is to determine whether or not WMA is lower in thermal diffusivity than HMA and hence cools more slowly under a given set of conditions. In the following section, we describe preliminary data analysis, which motivated our investigation into the main research question. The findings are then presented along with discussion that provides our perspective regarding the potential of WMA for use in long haul, nighttime, and late season paving.

### **Data Collected and Preliminary Analysis**

Asphalt mat temperatures during paving were recorded using an infrared camera from two projects in North Dakota (Table 1). These projects were among the first five WMA pilot projects of the state and were completed by two different paving contractors in September 2011 and June 2012, respectively. Overall, three different types of WMA were used, namely Advera, Evotherm, and foamed asphalt, which are among the most tried WMA additives and processes by the twenty northern states (Saboori et al. 2012). The project scope included blade patching on the existing HMA pavement, overlaying undivided two-lane rural highways with HMA and WMA, and compacting the overlays to two inches of thickness. Both projects used a windrow elevator attached to a paver and the same type of rollers in breakdown rolling and finish rolling, i.e., double steel drum rollers in vibratory and static modes, respectively. For intermediate rolling, a pneumatic tire roller was used for ND 15 paving, and a double steel drum, vibratory roller for ND 3 paving.

On each paving project, temperature recording was performed for two days for each type of asphalt mixes. The infrared camera, mounted on a tripod, was set up approximately five to ten feet away from the edge of the pavement. In most cases the camera was located where some physical marker, such as a survey stake or milepost, was present in the proximity. Temperature recording at a location was performed at the following intervals: (1) out of the haul truck (unloading), (2) immediately behind the paver (laydown), (3) immediately before the starts of breakdown rolling, intermediate rolling, and finish rolling, and (4) at conclusion of each stage of rolling. The average duration of temperature recording at a given location ranged from 25 minutes to 40 minutes, depending on the length of time from unloading to

finish rolling at that location. After finish rolling was completed, temperature recording was repeated at another location. The total number of temperature recording locations for each mix type varied from minimum 12 locations in the ND 3 HMA section to maximum 17 locations in the ND 15 Evotherm section.

Table 1

***Paving Projects Observed***

| <b>Project No.</b> | <b>Overall Length<sup>1</sup></b> | <b>HMA Control Section<sup>2</sup></b> | <b>WMA Trial Section<sup>2</sup></b> | <b>Grade of Asphalt Concrete</b> | <b>Aggregate</b>                  | <b>Compacted Thickness</b> |
|--------------------|-----------------------------------|----------------------------------------|--------------------------------------|----------------------------------|-----------------------------------|----------------------------|
| ND 15              | 21 miles                          | 3.5 miles                              | 3 miles                              | PG 58-28                         | Class 29 (HMA);                   | 2 inches                   |
| ND 3               | 18 miles                          | 3 miles                                | 2.5 miles (Advera)                   | PG 58-28                         | FAA <sup>3</sup> 43 (HMA and WMA) | 2 inches                   |

<sup>1,2</sup> Multiply 2 for equivalent lane-miles.

<sup>2</sup> Corresponds to the sections where temperature recording was performed.

<sup>3</sup> Fine aggregate angularity.

From each thermal image, the region of interest (i.e., freshly laid asphalt mat) was manually isolated so that the average temperature of asphalt mat surface could be determined. As a rule, the region of interest excluded the tapered edge of pavement because the edge did not receive compaction and no density measurements would be made of the edge that could be related to the mat temperature. All thermal pictures that were included for analysis presented here can be found in Song and Gao (2012). In both of the paving projects observed (see Table 1), WMA mat temperatures were less variable from sample location to sample location, in contrast to HMA temperatures that were higher at some locations and lower at others. Also WMA appeared to undergo less cooling; for ND 3 paving, on average WMA mat temperature dropped by 79°F (26°C) when finish rolling was complete, compared to 102°F (39°C) drop for HMA.

Despite this apparent difference in cooling between HMA and WMA, every lot paved (a 2,000 feet long and paver-wide section) achieved the same target density specified by NDDOT, or 91% of the daily average theoretical maximum density. In fact, in both ND 3 and ND 15 paving, average core density of HMA sections differed from that of WMA only by less than 1 percent point of the theoretical maximum density, or by 3 to 4 lb/ft<sup>3</sup> at most. HMA and WMA producing similar density results may be due to completion of breakdown rolling at above the cessation temperature 175°F (80°C) as the greatest density gains are made above 170°F with the breakdown roller (Schmitt et al. 2009).

Preliminary analysis showed some difference in cooling between HMA and WMA, but it could not answer whether WMA is different at all from HMA in thermal diffusivity. Because asphalt mat cooling is determined not just by thermal diffusivity but also by several uncontrollable conditions (e.g., ambient air temperature, wind speed), lesser cooling of WMA does not automatically translate into a low thermal diffusivity compared to HMA that cooled under different conditions – see Table 2; detailed weather records can be found in Ahmed (2015).

Table 2

***Environmental Conditions***

| Mix Type<br>(Project No.) | Average<br>Cooling | Average<br>Paving Cycle<br>Time* | Average Hourly<br>Air Temperature |      | Average Hourly Wind<br>Speed |        |
|---------------------------|--------------------|----------------------------------|-----------------------------------|------|------------------------------|--------|
|                           |                    |                                  | Low                               | High | Low                          | High   |
| Advera (ND 3)             | 79°F               | 36 min                           | 64°F                              | 71°F | 5 mph                        | 12 mph |
| HMA (ND 3)                | 102°F              | 33 min                           | 45°F                              | 77°F | 8 mph                        | 15 mph |
| Foamed (ND 15)            | 65°F               | 21 min                           | 57°F                              | 74°F | 6 mph                        | 11 mph |
| Evothrm (ND)              | 65°F               | 21 min                           | 47°F                              | 59°F | 13 mph                       | 21 mph |
| HMA (ND 15)               | 76°F               | 22 min                           | 66°F                              | 78°F | 12 mph                       | 22 mph |

\* From laydown to end of finish rolling

**Further Analysis and Discussion**

If WMA is lower in thermal diffusivity than HMA, WMA will cool more slowly under the same given conditions. But without knowing thermal diffusivity values, how can one tell? In answering this question, we followed two steps. First, we computed mat temperature of WMA with actual field conditions, except under a hypothesis that WMA has the same thermal diffusivity as HMA. For this computation, we used a PC-based software tool MultiCool due to Timm et al. (2001). MultiCool has pre-defined thermal diffusivity values for HMA and thus readily accommodated our hypothesis into prediction of WMA temperature under actual field conditions. As the second step, the resulting *average layer* temperature given by MultiCool was compared to actual *surface* temperature recorded at end of breakdown rolling – instead of end of intermediate rolling since the time until intermediate rolling ended varied significantly between the two projects. All input values into MultiCool, including actual air temperature and wind speeds, can be found in Ahmed (2015) as well as respective outputs.

An example comparison of WMA temperatures is illustrated in Figure 1, which shows three cooling curves for: a) actual mat *surface* temperature, b) MultiCool-computed *average layer* temperature given the above stated hypothesis, and c) mat *surface* temperature that would result if the hypothesis were true. Actual mat surface temperature (trend line *a* in Figure 1) was determined with the infrared camera, as described in the previous section. Average layer temperature (trend line *b* in Figure 1) was calculated by MultiCool under the same thermal diffusivity hypothesis. This MultiCool-predicted temperature is average temperature over mat thickness and corresponds to temperature of a point at some depth, not of a fixed point, beneath the mat surface, due to the transient nature of asphalt cooling during paving. From our physical intuitions, this average layer temperature (line *b* in Figure 1) should be higher than mat surface temperature (region *c* in Figure 1) that would result if the hypothesis were true. This is so because the interior mat was not directly exposed to the colder base or air and would not cool as fast as the upper mat surface that was subjected to cooling by winds as well as by lubricating water sprayed by the steel drum roller.

Comparing the two mat surface temperatures, one actual and the other expected under the hypothesis (*a* and *c* in Figure 1, respectively), actual cooling of WMA at the end of breakdown rolling is overestimated by an unreasonable amount, that is, more than 16°F or 5 minutes. We regard unreasonable overestimate as deviating from the actual by 10°F or more, which corresponds to difference of one minute or more in breakdown rolling time. Though arbitrary, we consider it to be too large to be explained by other sources of errors alone, e.g., infrared temperature measurements, thereby justifying rejection of the same diffusivity hypothesis for WMA. Table 3 counts the cases where the hypothesis was rejected according to the 10°F/one-minute criterion. It suggests that Advera mixes should have low thermal diffusivity values and will cool more slowly than HMA under given environmental conditions. On the other hand, foamed asphalt and Evotherm mixes did not provide as strong grounds that justify rejection of the hypothesis. That is, foamed asphalt and Evotherm mixes may be similar to HMA in thermal diffusivity and cool as fast as HMA will. Since the potential difference that some WMA can make in terms of cooling time or compaction time will depend on actual environmental conditions, we discuss later how much additional compaction time WMA potentially offers over HMA.

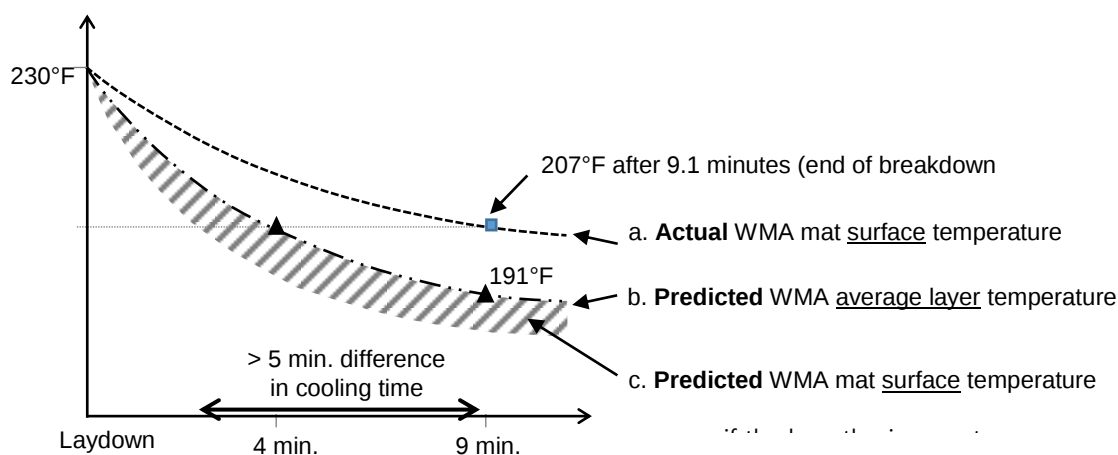


Figure 1: Sample WMA case of hypothesis rejection (not drawn to scale)

Table 3

### Number of Cases with Hypothesis Rejected

| Mix Type (Project No.) | Total No. of Cases | No. of Cases Rejected |
|------------------------|--------------------|-----------------------|
| Advera (ND 3)          | 10                 | 6 (60%)               |
| Evotherm (ND 15)       | 17                 | 4 (24%)               |
| Foamed (ND 15)         | 16                 | 2 (13%)               |

Our observation made of WMA's thermal diffusivity is based on temperature prediction using MultiCool. In order to assess whether the software tool predicts cooling of HMA consistent with the actual, we apply the same procedure to HMA. Note this is not to test any hypothesis, but to check validity of MultiCool computation in view of actual HMA temperature although this has been previously done by others (Chadbourn et al. 1998). Figure 2 illustrates an example result. As discussed earlier, physical intuitions

indicate that average layer temperature (trend line *b* in Figure 2) predicted with actual field conditions should be higher than the corresponding surface temperature (area *c* in Figure 2). For the case shown in Figure 2, this means that MultiCool-predicted, HMA surface temperature (area *c* in Figure 2) falls on the right side to include actual HMA surface temperature (trend line *a* in Figure 2). In fact, this was true for 23 out of 27 HMA cases (85%); as per the same 10°F/one-minute rule, only four cases (15%) involved significant incompatibility with actual temperature. As such, we consider MultiCool-prediction of HMA temperature using the pre-set thermal diffusivity values of HMA to be consistent with actual HMA temperature. In a sense, our investigation using MultiCool has discriminative power to not only reject a certain WMA as not HMA-like but also accept HMA as HMA-like. Because of a small sample size, we however do not attach any statistical significance to the conclusion drawn about thermal diffusivity of WMA relative to that of HMA.

In summary, a certain WMA mixes may have a lower thermal diffusivity value than HMA. Our analysis indicated that Advera mixes used in ND 3 paving should have a lower thermal diffusivity value than HMA mixes used in ND 3 paving. Concluding otherwise would unreasonably invalidate actual recorded temperature or MultiCool-calculated temperature. On the other hand, our analysis did not yield as strong justification that supports rejection of the same thermal diffusivity hypothesis for Evotherm mixes and foamed asphalts used in ND 15 paving. Within our limited data, Evotherm and foamed asphalt mixes are rather similar to HMA in their ability to keep heat.

If some WMA is low in thermal diffusivity, it will cool more slowly when paved under the same conditions as HMA would be paved. This brings us to the point of departure for our investigation: How much additional compaction time would a slow rate of cooling of WMA translate into? As stated earlier, actual difference WMA can make in terms of compaction time will depend on initial and boundary conditions (i.e., laydown temperature, air temperature, winds, etc.). Depending on the conditions during paving, a given WMA will exhibit varying cooling rates – after all, cooling rate in itself is not a material property – and hence any additional compaction time will also vary. This observation, along with lack of thermal diffusivity values of WMA, limits our ability to answer the question about additional compaction time. Here we give one suggestive example.

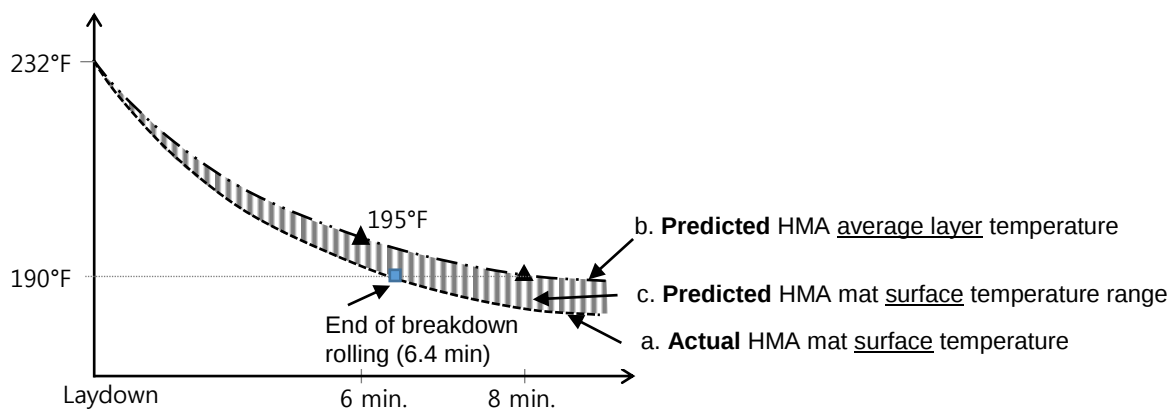
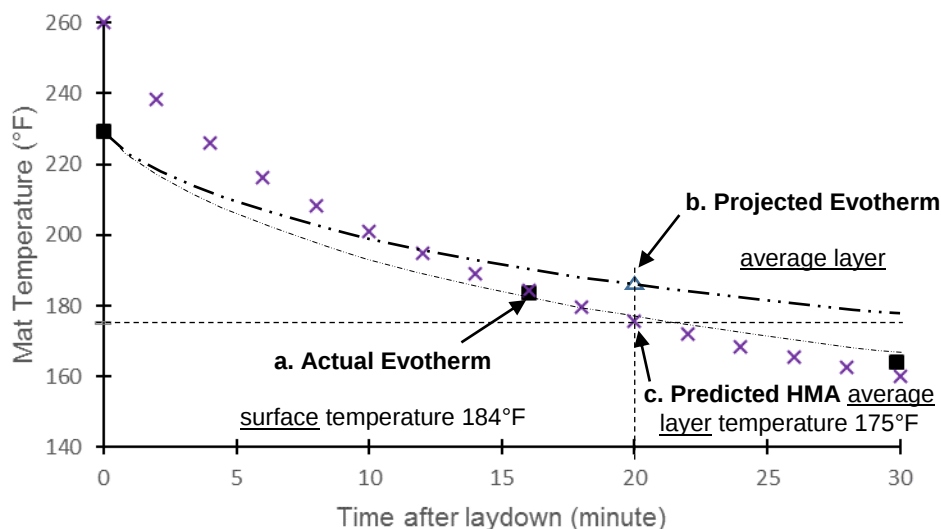


Figure 2: Sample HMA case (not drawn to scale)



**Figure 3:** Comparing cooling of WMA (actual) and HMA (assumed) under the same environmental conditions (existing base 98°F, air 59°F, wind speed 17 mph, and partly cloudy)

Figure 3 depicts one of the four Evotherm cases where the same thermal diffusivity hypothesis was rejected and Evotherm mixes could be considered to be low in thermal diffusivity. Three kinds of asphalt temperature are plotted in Figure 3: a) actual mat surface temperature of Evotherm mix; b) estimated, average layer temperature of Evotherm; and c) MultiCool-predicted average layer temperature of HMA assuming 260°F at laydown. Note all three different temperatures in Figure 3 are due to the same set of environmental conditions under which an Evotherm mix was actually paved. *If* HMA were paved instead of Evotherm mix, its average layer temperature (c in Figure 3) 20 minutes after laydown would be 175°F according to MultiCool's prediction. We project average layer temperature of Evotherm mix (b in Figure 3) at the same time to be 190°F. This projection is made based off the trend line of actual surface temperature of Evotherm mix (denoted by marker 'a' in Figure 3) and considering the observation (Tegeler and Dempsey 1973): "the surface temperatures of a 2-inch layer having an average temperature of 175°F on an average summer day [in Illinois] would be somewhere between 140°F and 160°F." Comparing in terms of average layer temperature, Evotherm mix (b in Figure 3) would be 15°F higher than HMA (c in Figure 3) and take about another 10 minutes to cool to the same temperature 175°F. Therefore, slow cooling of Evotherm in the case of Figure 3 would allow additional 10 minutes of compaction, provided Evotherm mixes at or above 175°F of average layer temperature can be compacted properly, which was true for our data – where the required density was obtained, the minimum mat surface temperature at end of intermediate rolling was around 160°F for Evotherm as well as HMA sections, or 175°F in terms of average layer temperature.

We however note from our field observations that 20 minutes from laydown is already sufficient time for HMA to make a total of 12 or more passes of breakdown and intermediate rolling and achieve required density. This suggests that expanded time window for compaction, potentially offered by use of WMA, may not be practically necessary or beneficial unless asphalt paving is to be done under the limiting field conditions that normally prohibit HMA paving. In such limiting conditions, a certain minimum number of roller passes can still be made to compact WMA properly. We conclude our discussion with a question that is beyond the scope of this paper and merits further study: Under what field conditions can WMA but

not HMA be compacted properly?

## Conclusions and Recommendations

Based on our analysis of the limited data from two asphalt paving projects, we conclude that a certain WMA mixes are lower in thermal diffusivity and will cool more slowly than HMA under similar field conditions. Relatively slow cooling of WMA may provide for sufficient compaction time that is not feasible with HMA under adverse field conditions. Thus, WMA does offer the potential for use in nighttime and late season paving although other characteristics of WMA, such as rutting resistance to expected traffic loads, may dominate the decision whether or not to use WMA for a given job. However, in order to achieve proper density with WMA under adverse field conditions, the same minimum number of roller passes as required for HMA may be required and should be provided within a relatively narrow time window. If WMA is to be compacted to a higher density than normally required of HMA, a greater number of roller passes would be required, which actual field conditions may or may not permit. Also, “soft” environmental impacts relative to WMA applications in adverse conditions could be explored. For example, with available compaction time known or assumed, the required number of passes to achieve specified density given the paver and roller speed could be estimated using discrete event simulation.

To help to make an informed decision regarding application of WMA in adverse conditions, further study is recommended to determine under what particular field conditions WMA but not HMA allows for sufficient time to compact properly. This will require the knowledge of thermal diffusivity values of WMA. Although thermal diffusivity values of WMA can be estimated through experimentation similarly to Chadbourn et al. (1998), a less time-consuming method could be developed that solves the inverse heat transfer problem for the unknown thermal diffusivity constant, given mat surface temperature measurements. Such a method would permit non-intrusive field determination of thermal diffusivity of asphalt mixes, whether WMA or HMA. Once the thermal diffusivity value has been determined for a given mix, the underlying computational model of asphalt cooling could be used to predict available compaction time in reference to temperature of the mat surface. Although “there is nothing to compact at a point on the surface” (Corlew and Dickson 1968), predicted mat surface temperature can be readily validated with actual mat surface temperature, and paving operations can be adjusted accordingly. One challenge will be to account for additional cooling on the surface due to lubricating water from the roller, which may alter the governing equation to be a non-linear differential equation.

## References

- Ahmed, Syed O. (2015). *Comparing Cooling of Warm Mix to Hot Mix Asphalts*. M.S. Thesis, North Dakota State University.
- APEC (Asphalt Pavement Environmental Council) (2000). *Best Management Practices to Minimize Emissions during HMA Construction*, EC-101.
- Bell, C.A., Hicks, R.G., and Wilson, J.E. (1984). Effect of Percent Compaction on Asphalt Mixture Life. *Placement and Compaction of Asphalt Mixtures*, ASTM Special Technical Publications 829, 107-130.

- Chadbourn, B.A., DeSombre, R.A., Newcomb, D.E., Luoma, J.A., Voller, V.R., and Timm, D.H. (1998). *An Asphalt Paving Tool for Adverse Conditions*, Report No. MN/RC 1998-18, Minnesota Department of Transportation.
- Chang, C., Chang, Y., Chen, J. (2009). Effect of Mixture Characteristics on Cooling Rate of Asphalt Pavements. *Journal of Transportation Engineering*, ASCE, 135 (5), 297-304.
- Corlew, J.S., and Dickson, P.F. (1968). Methods for Calculating Temperature Profiles of Hot Mix Asphalt Concrete as Related to the Construction of Asphalt Pavement. *Asphalt Paving Technology*, Proceedings of Association of Asphalt Paving Technologists, 37, 101-140.
- Dickson, P.F., and Corlew, J.S. (1970). Thermal Computations Related to the Study of Pavement Compaction Cessation Requirements. *Asphalt Paving Technology*, Proceedings of Association of Asphalt Paving Technologists, 39, 377-403.
- Hughes, C.S. (1989). *NCHRP Synthesis of Highway Practice 152: Compaction of Asphalt Pavement*, Transportation Research Board, Washington, D.C.
- Jordan, P.G., and Thomas, M.E. (1976). *Prediction of Cooling Curves for Hot-Mix Paving Materials by a Computer Program*, Transport and Road Research Laboratory Report 729, 31 p.
- Kristjansdottir, O., Muench, S.T., Michael, L., Burke, G. (2007). Assessing Potential for Warm-Mix Asphalt Technology Adoption, *Transportation Research Record* 2040, 91-99.
- Powers, D.L. (2006). *Boundary Value Problems and Partial Differential Equations*, Elsevier Academic Press.
- Saboori, A., Abdelrahman, M., and Ragab, M. (2012). *Warm Mix Asphalt Processes Applicable to North Dakota*, Research Report submitted to the North Dakota Department of Transportation.
- Scherocman, J.A. (2006). Compaction of Stiff and Tender Asphalt Concrete Mixes. *Transportation Research E-Circular*, Issue E-C105, 69-83.
- Schmitt, R.L., Bahia, H.U., Johnson, C., and Hanz, A. (2009). *Development of Recommendations for Compaction Temperatures in the Field to Achieve Density and Limit As-Built Permeability of HMA in Wisconsin*, Wisconsin Department of Transportation; Federal Highway Administration, 2009, 146 p.
- Song, J., and Gao, J. (2012). *Field Investigation of Warm Mix Asphalt Temperatures*, Research Report submitted to the North Dakota Department of Transportation.
- Tegeler, P.A. and Dempsey, B.J. (1973). A Method of Predicting Compaction Time for Hot-Mix Bituminous Concrete. *Asphalt Paving Technology*, Proceedings of Association of Asphalt Paving Technologists, 42, 499-523.
- Timm, D H, Voller, V R, Lee, E B, Harvey, J (2001). Calcool: A Multi-Layer Asphalt Pavement Cooling Tool For Temperature Prediction During Construction. *International Journal of Pavement Engineering*, 2 (3), 169-185.
- Von Quintus, H.L., Rao, C., Minchin Jr., R.E., Nazarian, S., Maser, K.R., Prowell, B.D. (2009). *NDT Technology for Quality Assurance of HMA Pavement Construction*, NCHRP Report 626, 120 p.

Willoughby, K.A., Mahoney, J.P., Pierce, L.M., Uhlmeier, J.S., Anderson, K.W., Read, S.A., Muench, S.T., Thompson, T.R., and Moore, R. (2001). *Construction-Related Asphalt Concrete Pavement Temperature Differentials and the Corresponding Density Differentials*, Washington State Department of Transportation; Federal Highway Administration, 163 p.