Application of AHP and Fuzzy AHP to Decision-Making Problems in Construction

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The approach to selecting key factors has been a critical matter in the research of construction management for a long period due to facts that many construction problems often require the consideration of multiple factors at the same time and depend on subjective judgments made by experts in their fields of interests. For that reason, the analytic hierarchy process (AHP) method has been widely used in decision-making situations where multiple factors have to be considered at the same time. It has been believed that the AHP is suitable for assisting decision-makers surrounded by a number of factors. In recent times, a derivative of the AHP, called the fuzzy AHP, has been deployed by mixing the principles of the AHP with a fuzzy set theory. This new approach increasingly widens its application areas in construction management. Little research, however, has been performed with respect to the comparability of calculated priority weight vectors by the new approach, fuzzy AHP, with those by the traditional AHP. Thus, this article mainly focuses on the comparison of two AHP approaches by examining weight vectors generated from a previous case study. The major findings and results of this study are presented and discussed.

Key Words: Analytic Hierarchy Process (AHP), Fuzzy AHP, Criteria weightings, Decision-making

Introduction

The approach to selecting main factors has been a critical matter in the research of construction management for a long period, mainly because many construction problems often require the consideration of multiple factors at the same time and depend on subjective judgments made by experts in their fields of interests. For that reason, the analytic hierarchy process (AHP) method has been widely applied in decision-making situations since Thomas Saaty introduced the method in 1980 (Saaty, 1980). The method has been implemented successfully to calculate priority weights in various decision-making problems. Previous studies may include the application of the AHP to the evaluation of advanced construction technologies (Skibniewski and Chao, 1992), the application to the determination of bid-markup sizes (Lee and Chang, 2004), the selection of construction equipment for construction projects (Shapira and Goldenberg, 2005), the evaluation of projects and proposals (Su et al., 2006; Bertolini et al., 2006), and the assessment of asset management methods (Cooksey et al., 2011).

Recently, a derivative of the AHP, called the fuzzy AHP, has been deployed by mixing the principles of the AHP with a fuzzy set theory. A main reason for the need of the new approach is that a traditional AHP is not effective in dealing with uncertainty when decision makers choose a scale from a given fundamental scale of 1 to 9. To reflect the uncertainty, decision makers require more flexible scales by using fuzzy membership functions and linguistic variables, e.g. good or poor, rather than using deterministic or crisp values (Soroor et al., 2012). This new method has been applied to numerous areas of the construction industry with similar purposes as the conventional AHP method. It seems that the fuzzy AHP was studied a lot in the area of risk management among all areas due to the fact that it is more advantageous to provide flexible scales from fuzzy membership functions (Khazaeni et al., 2012; Zeng et al., 2007; Zhang and Zou, 2007).

Despite the increasing popularity of the fuzzy AHP, little research has been performed with respect to the comparability of calculated priority weight vectors by the new approach, fuzzy AHP, with those by the traditional AHP. Therefore, this paper mainly focuses on the comparison of two AHP approaches by analyzing weight vectors generated from a previous case study performed by Lee and Chang (2004) and discussing the similarity or dissimilarity of the results. The results of priority weights obtained from this empirical study not only present the most comparable fuzzy AHP approach, but also enlighten the degree of differences between two AHP approaches.

Traditional AHP Method

The popular use of the AHP places in the ease of use, the flexibility to integrate various factors with quantitative and qualitative nature, and extensive applications and publications. These advantages have been proved successfully in many multifaceted problems. The method was developed to assist decision-making problems involving a number of relevant factors. The application of the AHP to such problems requires the execution of two phases: a hierarchy design and its evaluation (Vargas, 1990). The design of hierarchies and their evaluations greatly require the experience and knowledge of the problem at hand. It means that a lot of attention has to be paid to those phases because many judgments are made subjectively. Two experts might design two different hierarchies to the same problem. It is important that a decision maker represents the problem meticulously when structuring a hierarchy. The evaluation of the hierarchy is also affected by subjective judgments of decision makers.

The evaluation phase is based on the concept of pair-wise comparisons. The elements in a level of the hierarchy are compared in relative terms as to their importance or contribution to a given criterion. Thomas Saaty prepared the importance scale for the comparisons ranging from equally important to extremely important as shown in Table 1. In general, a hierarchy has three levels, i.e. a goal, criteria, and alternatives. A goal is placed at the top level, and criteria are placed at the middle level. Criteria may have multiple levels depending on the complexity of the given problem. At the bottom level, alternatives are located. First, criteria are prioritized by the pair-wise comparisons. This process of comparison yields a relative scale of measurement of the priorities or weights of the elements. And then, the alternatives at the lowest level are compared corresponding to each criterion. Finally, the priorities from the level of criteria and the level of alternatives are aggregated additively making the total score. The more explanations on the calculations are not provided here due to a page limit, but can be found in the previous studies (Saaty, 1980; Skibniewski and Chao, 1992; Shapira and Goldenberg 2005).

Table 1

AHP Fundamental Scale

Intensity of Importance	Definition
1	Equal importance
3	Moderate importance of one over another
5	Essential or strong importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8	Intermediate values

Fuzzy AHP Approach

A fuzzy AHP approach was developed from a fuzzy set theory first introduced by Zadeh (1965). He developed the theory to represent ambiguity that cannot be explained by a usual mathematical sense of terms, e.g. “the class of tall people” or “the group of beautiful women.” It has been known that the fuzzy set theory is quite effective when handling problems in which there are no sharp boundaries and precise numbers. Furthermore, fuzzy numbers are not like rigid mathematical terms and equations, but are close to human natural language.

A fuzzy set is different from a crisp set. Fuzzy numbers can be any real number in the interval [0, 1] by fuzzy membership functions, whereas crisp sets only allow either 0 or 1. As the fuzzy number is close to one, the degree of membership of the number is higher. In many applications, triangular fuzzy numbers (TFNs) were used due to their computational simplicity and ability to promote representation and information processing in a fuzzy environment (Khazaeni et al., 2012). A triangular fuzzy number, $\tilde{A}$, on R can be denoted as (s, t, u), and its membership function can be defined as follows.

$$\mu_{\tilde{A}(x)} = \begin{cases} 0, & x \leq s \\ \frac{x-s}{t-s}, & s \leq x \leq t \\ \frac{x-u}{t-u}, & t \leq x \leq u \\ 0, & otherwise \end{cases} \quad (1)$$

When there are two triangular fuzzy numbers, $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$, their operational laws are as follows.

$$\tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1+b_1, a_2+b_2, a_3+b_3) \quad (2)$$

$$\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = (a_1b_1, a_2b_2, a_3b_3) \quad (3)$$

$$\tilde{A}^{-1} = (1/a_3, 1/a_2, 1/a_1) \quad (4)$$

$$\lambda \otimes \tilde{A} = \lambda \otimes (a_1, a_2, a_3) = (\lambda a_1, \lambda a_2, \lambda a_3) \ (\lambda > 0, \lambda \in R) \quad (5)$$

Figure 1 shows two triangular fuzzy numbers, $\tilde{A}$ and $\tilde{B}$, to illustrate the operations of fuzzy numbers. The fuzzy number of $\tilde{A}$ can be represented as (1, 2, 3) and the fuzzy number of $\tilde{B}$ can be expressed as (2, 3, 4). The addition of two numbers, $\tilde{A} \oplus \tilde{B}$, generates a new fuzzy number of (3, 5, 7) based on the above equation.

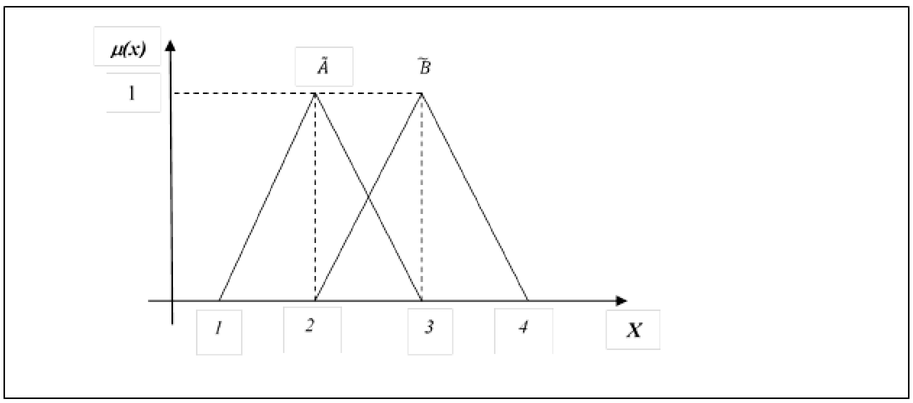

Figure 1: Triangular Fuzzy Numbers

The main objective of this research study is to make a comparison with weight results obtained by the traditional AHP through a case study completed. To do this, comparable fuzzy AHP methods should be first selected carefully. Different fuzzy AHP approaches were proposed in construction management as indicated in the introduction, but they are not applicable directly for this study. From the literature review, it was found that not all previous studies were applicable since they did not apply the fundamental scale by Saaty, involve multiple decision-makers, or apply the fuzzy AHP with weight determination purpose. Also, it was found that many fuzzy AHP studies are based on the extent analysis proposed by Chang (1996) by various authors. While there is some similarity in the general concepts, detailed application parts show variations between fuzzy AHP methods utilized before, especially in the fuzzy membership functions and the way of weight aggregations. In the fuzzy AHP, the weight importance of decision criteria are evaluated by using a modified fundamental scale expressed as linguistic terms. There were two types of fuzzy fundamental scales developed for evaluating criteria as shown in Tables 2 and 3 (Yazdani-Chamzini and Yakhchali, 2012; Khazaeni et al., 2012). Another variation observed was how to aggregate multiple weights. In group decision-making situations with *z* experts, the importance of evaluation criteria can be calculated in two different ways. The first way is as follows.

Weight Aggregation I (WAI):

$$\widetilde{w_{ij}} = (Lw_{ij}, Mw_{ij}, Uw_{ij}) \quad (6)$$

$$Lw_{ij} = min_z\{Lw_{ijz}\}, Mw_{ij} = \frac{1}{z}\sum_{z=1}^{Z} Mw_{ijz}, Uw_{ij} = max_z\{Uw_{ijz}\} \quad (7)$$

Where, $\widetilde{w_{ij}}$ is the triangular fuzzy weight of the *i*th criterion in comparison with the *j*th criterion.

The second way is based on fuzzy arithmetic operations and is as follows.

Weight Aggregation II (WAII):

$$Lw_{ij} = \frac{1}{z}\sum_{z=1}^{Z} Lw_{ijz}, Mw_{ij} = \frac{1}{z}\sum_{z=1}^{Z} Mw_{ijz}, Uw_{ij} = \frac{1}{z}\sum_{z=1}^{Z} Uw_{ijz}, \quad (8)$$

Research Methodology

As explained before, there are various ways to apply a fuzzy AHP method to calculate priority weights, and the weight results could be different depending on specific variables you choose. From the literature review, two kinds of fuzzy fundamental scales were identified, and two kinds of weight aggregations were utilized. Therefore, four different fuzzy AHP approaches were tested in this paper by mixing fuzzy fundamental scales and weight aggregations. It means that priority weights are calculated from total five approaches including the traditional AHP for a comprehensive comparison (see Table 4).

Table 2

Fuzzy Fundamental Scale I

Linguistic Term	Fuzzy Number	Triangular Fuzzy Scale	Reciprocal Fuzzy Scale
Equally important	$\tilde{1}$	(1, 1, 1)	(1, 1, 1)
Intermediate value	$\tilde{2}$	(1, 2, 3)	(1/3, 1/2, 1)
Moderately important	$\tilde{3}$	(2, 3, 4)	(1/4, 1/3, 1/2)
Intermediate value	$\tilde{4}$	(3, 4, 5)	(1/5, 1/4, 1/3)
Strongly important	$\tilde{5}$	(4, 5, 6)	(1/6, 1/5, 1/4)
Intermediate value	$\tilde{6}$	(5, 6, 7)	(1/7, 1/6, 1/5)
Very strongly important	$\tilde{7}$	(6, 7, 8)	(1/8, 1/7, 1/6)
Intermediate value	$\tilde{8}$	(7, 8, 9)	(1/9, 1/8, 1/7)
Extremely important	$\tilde{9}$	(8, 9, 10)	(1/10, 1/9, 1/8)

Table 3

Fuzzy Fundamental Scale II

Linguistic Term	Fuzzy Number	Triangular Fuzzy Scale	Reciprocal Fuzzy Scale
Equally important	$\tilde{1}$	(1, 1, 1)	(1, 1, 1)
Intermediate value	$\tilde{2}$	(1, 2, 4)	(1/4, 1/2, 1)
Moderately important	$\tilde{3}$	(1, 3, 5)	(1/5, 1/3, 1)
Intermediate value	$\tilde{4}$	(2, 4, 6)	(1/6, 1/4, 1/2)
Strongly important	$\tilde{5}$	(3, 5, 7)	(1/7, 1/5, 1/3)
Intermediate value	$\tilde{6}$	(4, 6, 8)	(1/8, 1/6, 1/4)
Very strongly important	$\tilde{7}$	(5, 7, 9)	(1/9, 1/7, 1/5)
Intermediate value	$\tilde{8}$	(6, 8, 9)	(1/9, 1/8, 1/6)
Extremely important	$\tilde{9}$	(7, 9, 9)	(1/9, 1/9, 1/7)

Table 4

Five Approaches for Comparison

Approach	Approach 1 (APRH1)	Approach 2 (APRH2)	Approach 3 (APRH3)	Approach 4 (APRH4)	Approach 5 (APRH5)
Types of AHP	Traditional AHP (Lee and Chang (2004))	Fuzzy AHP (Mixture of FFSI and WAI)	Fuzzy AHP (Mixture of FFSI and WAII)	Fuzzy AHP (Mixture of FFSII and WAI)	Fuzzy AHP (Mixture of FFSII and WAII)

A case study performed by Lee and Chang (2004) was adopted for an empirical study by considering data availability. A traditional AHP method was applied to determine priority weights among decision factors that have impact on the bid-markup size in microtunneling projects. Figure 2 presents a decision hierarchy where bidding criteria are divided into two levels, classification and subclassification. The classification level includes company, project, and microtunneling factors. Each classification factor has a number of subclassification factors. There are five subclassification factors under the company factor. And, there are six subclassification factors linked to the project and microtunneling factors, respectively. To compute weight vectors, total four pairwise comparison matrices were evaluated by using five approaches, APRH1 to APRH5, according to either a fundamental scale or a fuzzy fundamental scale. Pairwise comparison matrices prepared by crisp values were converted to fuzzy pairwise comparison matrices by the above fuzzy membership functions. For example, if a factor A is three times more preferred (moderate importance) to a factor B, then a fuzzy scale of (2, 3, 4) is assigned by the first fuzzy fundamental scale, and a fuzzy scale of (1, 3, 5) is allocated by the second fuzzy fundamental scale. This fuzzification process had been performed to all levels of a decision hierarchy, i.e. classification, company, project, and microtunneling levels.

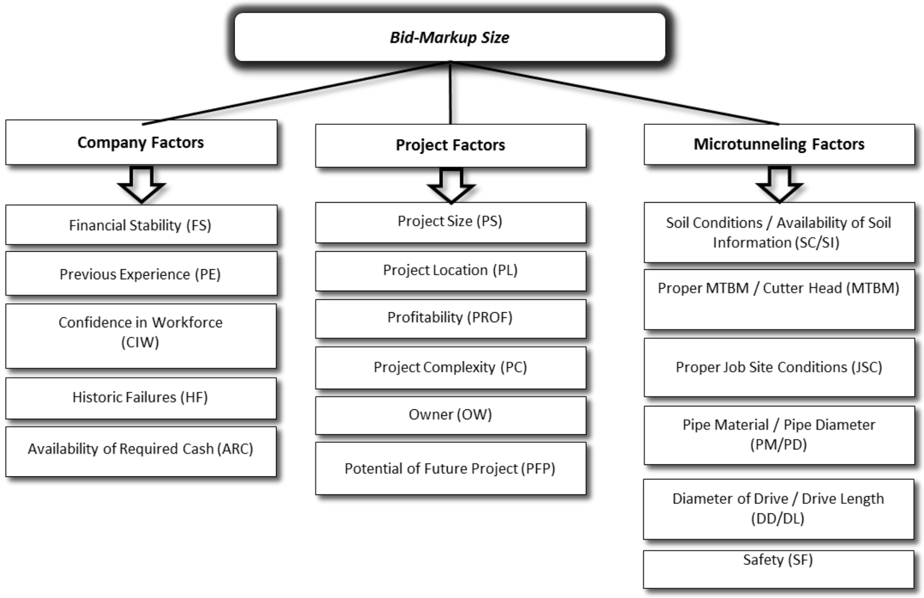

Figure 2: Decision Hierarchy for the Determination of Bid-markup Size (MTBM: Microtunneling boring machine)

Results and Discussion

Priority weight vectors obtained from the traditional AHP and the fuzzy AHP are computed and discussed in this section to make a comparison of five approaches. The degree of similarity or difference in comparison with the traditional AHP can be identified through the comprehensive comparison, majorly based on calculated priority weights.

Priority weights were calculated to the four pairwise comparisons in the classification and subclassification levels. The weights by the fuzzy AHP were generated by following computation procedures proposed by Chang (1996) that was determined suitable for this study.

In addition, the root mean square error (RMSE) was employed as a quantitative measure to identify the degree of deviations from target data, i.e. data obtained from the traditional AHP. The RMSE value can be calculated by using Equation (9), and was calculated to every decision factor by all applied fuzzy methods.

$$\textbf{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - x_i^T)^2}{n}} \qquad (9)$$

Where, X_i: Weight by the fuzzy AHP, X_i^T: Weight by the traditional AHP, and n: Number of factors in a matrix.

Table 5 shows the RMSE values of four fuzzy approaches, and Figure 3 presents the trend of their error rates graphically. Although the amount of deviations seems to be variable between fuzzy AHP methods, APRH5 presented the lowest RMSE value in total among four methods. And, APRH2 and APRH4 followed the APRH5, while displaying similar performances in the evaluation of matrices. Unlike the other three approaches, the error rates of APRH3 were proportional to the number of factors in a matrix as indicated in Figure 2. The first pairwise matrix of “classification” contains three factors, the second matrix of “company” involves five factors, and the third and fourth ones, “project” and “microtunneling”, require the comparison of six factors. While the APRH3 showed the best performance in the evaluation of three factors, it revealed the worst performances in the matrices of “Company”, “Project”, and “Microtunneling.”

Although two approaches generated comparable results in some cases, they cannot make the same results mainly because they process pairwise comparison matrices in a different way. In a traditional AHP method, each pairwise matrix is evaluated individually, and then weight vectors are combined together by a geometric mean. But, in a fuzzy AHP method, all pairwise matrices are combined first by using a predetermined weight aggregation, and then a single weight vector is calculated in the end.

Table 5

Root Mean Square Errors of Fuzzy AHP

Approach	Classification	Company	Project	Microtunneling	Sum
APRH2	0.0225	0.0519	0.0779	0.0259	0.1782
APRH3	0.0037	0.0746	0.1082	0.1901	0.3766
APRH4	0.0220	0.0538	0.0884	0.0295	0.1937
APRH5	0.0159	0.0232	0.0443	0.0313	0.1147

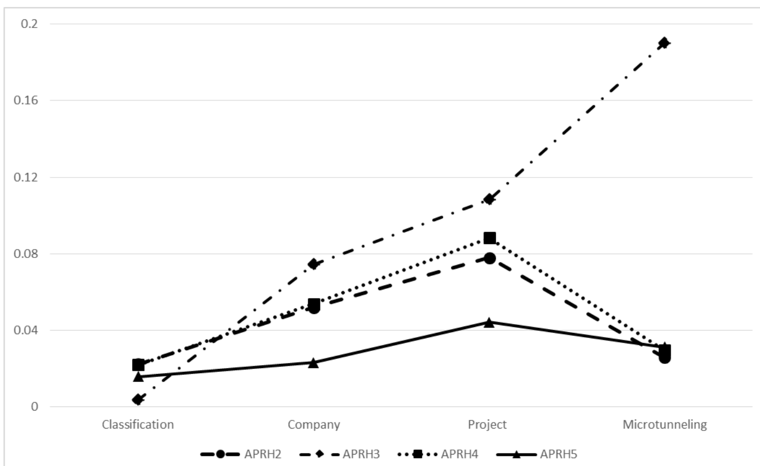

Figure 3: Comparison of Root Mean Square Errors

Conclusions

This research project started from the curiosity about how much comparable the fuzzy AHP is with the traditional AHP when calculating priority weights under multi-attribute decision-making situations. To accomplish the objective that was not considered well before, previously applied fuzzy AHP methods were examined carefully to select the best fitting method for further comparison. Four different approaches were analyzed in this study by varying fuzzy fundamental scales and weight aggregations. And, they were experimented to identify which method produces the most comparable results with the traditional AHP. As a result, it was found that priority weights could be significantly different depending on the design of fuzzy membership functions and the way of aggregating experts' opinions. In general, error rates tended to increase as more factors were involved in an evaluation matrix. On the whole, APRH5 showed the best performance when compared with a traditional AHP method in terms of priority weights and root mean square errors. APRH2 also displayed a good performance in the evaluation process. But, APRH3 generated the worst performance since it showed higher error rates and too many zero weights. The main limitation of the study was to utilize one case study. The author made an effort to search a lot of previous studies, but could not obtain real case study data. Almost all previous studies attempted to develop a generic model and only showed an illustration example with fictitious numbers. Although one case study was analyzed in this article, it is a real case study where all numbers are real captured from surveys. A decision hierarchy contains multiple factors, 20 factors, from which weight vectors were drawn.

Based on the results of two approaches, a few comparisons can be made as practical guidelines for readers who consider either approach. When the level of uncertainty in a problem on hand is low in determining a scale number, the traditional AHP can be favorable. If not, the fuzzy AHP needs to be considered to apply fuzzy flexible scales. Regarding the ease of use, the traditional AHP seems to be easier to use mainly because it deals with only one number at a time. Fuzzy numbers are expressed with multiple numbers, e.g. three or four. If users are not familiar with fuzzy number operations, e.g. addition or multiplication, they can make errors in the middle of calculations.

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