Case Study of a Decision-Making/Support Process to Schedule Multiple Highway Construction Projects

Arlys Silva
The University of Southern Mississippi
Hattiesburg, MS

Tulio Sulbaran
The University of Southern Mississippi
Hattiesburg, MS

The scheduling of numerous construction projects considering several factors is an important challenge faced by construction companies, particularly road builders. Road builders need to make decision to identify the best possible schedule to complete multiple constructions while considering multiple factor such as maximizing profit, reducing risk, and resource constraints among many other factors.

The decision-making process is very complex and should consider both quantitative and qualitative elements of all projects. Thus, this paper focuses on decision-making challenges faced by construction companies to schedule multiple projects by implementing a systematic decision-making framework using Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to help construction companies schedule several highway construction projects.

This paper presents a case study that will assist the company schedule the projects according to constraints, priorities and information of that particular company. The case study in this paper includes Company X, with 13 projects in Alabama and Florida. The company representative provided the 13 criterion that were important to make the decision. Additionally, the company also provided both the quantitative and qualitative data for the 13 established criteria and the 13 projects.

The results of the case study suggest that the systematic approach used could support the decision-making process of the construction companies and be very beneficial. The systematic approach can help decision-makers elect the best schedule alternative and remove any uncertainties which usually cloud decision-makers when the information is too large to keep track of.

Keywords: TOPSIS, Decision-Making, Case Study, Scheduling, Projects

Introduction

The ability of road builders to complete a project on time is affected by many factors such as bad weather, subcontractor issues, equipment breakdowns, labor availability, and unforeseen events [3]. A study conducted by the State Highway Agencies (SHAs) showed that the most common delays in construction were due to: a-utility relocation, b-utility conflicts, c-plan and specification errors, d-weather, e- permitting problems, f-changed orders, g-environmental delays, h-design issues, i-not enough workers/equipment, and j-condition of the project surface [6]. However, a well-planned schedule that captures all construction company projects will give the company better opportunities to overcome these delaying factors and others.

The case study presented in the paper uses the framework for decision support system based on the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), which demonstrates how a company can organize and rank highway projects based on their particular constraints, priorities and information.

TOPSIS is a Multi Criteria Decision Making (MCDM) method used to make better decisions amongst desirable and non-desirable alternatives. In the case of construction companies, alternatives are the scheduling of construction projects which are judged based on a set of criteria established by the leaders of the company. TOPSIS is a method that allows the decision-maker to obtain a solution that is closest to the positive ideal solution ($PIS - A^+$), and

farthest from the negative ideal solution ($PIS – A^-$). In fuzzy TOPSIS and for this paper, the positive ideal solution will be a number closest to 1 and the negative ideal solution will be a number closest to 0; thus, the ranking is done from 1 – 0 (1 being first place and 0 being last place). Since the data in the decision matrix is going to be normalized in the range between 0 – 1, the ideal solution for the case study in this paper will be $PIS – A^+$ (1, 1, 1) and the worst solution will be $NIS – A^-$ (0, 0, 0). There is a possibility that some of the ranking might have the same value in the project list being evaluated. For this paper, this will mean that the projects with the same value or close to the same value are projects that might have to be started at the same time or need to be scheduled concordantly.

The selection criteria include a mix of quantitative and qualitative constraints and priorities of the projects; the particular company and priorities are presented in a linguistic manner that will allow the company representatives to easily and clearly convey their expertise. The linguistics representation will help the decision-makers express constraints and priorities that are too complex and cannot be demonstrated with any numerical value [7]. This linguistics representation will then be transformed using a scale table of importance represented by triangular fuzzy number characteristics in order to rank each project. One example of a qualitative criterion will be "Risk of the project", where the construction representatives could provide their opinion using one of the following linguistic representations: Very Low, Low, Medium Low, Medium, Medium High, High, and Very High. [4].

Case Study Scheduling Multiple Highway Construction Project

An interview was conducted of a Project Manager of actual completed highway construction projects performed by Company X in Florida and Alabama. The interview resulted in the thirteen criteria below to evaluate projects' importance. It is worth noting that this set of criteria can be modified according to each particular company's operations.

- *C1 - Budgeted cost of the project.*
- *C2 - Bonus amount for early completion (The higher the amount per day, the better).*
- *C3 - Amount of liquidated damages depending on contract amount.*
- *C4 - Total contract days.*
- *C5 - Number of crews needed to complete the project.*
- *C6 - Number of equipment needed to complete the project.*
- *C7 - Number of subcontractors to complete the project*
- *C8 - Total asphalt tonnage for the project (The higher the tonnage, the sooner the company needs to start and finish the project. This will help with dump truck availability).*
- *C9 - Distance of the project in miles from the asphalt plant (The longer the project in miles, the more important it is to the company. The amount of trucks used will be higher).*
- *C10 - Level of complexity of the project:*
  - *Roadway milling & resurface (VL)*
  - *Asphalt excavation & resurface (L)*
  - *Clearing & grubbing, excavation, storm drainage, and roadway milling (ML)*
  - *Clearing & grubbing, excavation, storm drainage, concrete work, roadway milling, signage, and performance sod (M)*
  - *Clearing & grubbing, excavation, bridge work (removal & installation) storm drainage, concrete work, roadway milling, signage, and performance sod (MH)*
  - *Clearing & grubbing, excavation, foundation backfill, roadbed stabilization, bridge work (removal & installation), storm drainage, concrete work, roadway milling, signage, and performance sod (H)*
  - *New roadway construction including bridge work (VH)*
- *C11 – Cost risk of the project. Projects with no profit are riskier than projects with profit. Room for error is very low.*
- *C12 – Environmental risk (High or low level of erosion control that needs to be performed. The more the erosion control, the higher the chances of delay and not completing by the completion date due to the weather).*
- *C13 – Priority of project due to special circumstances (This could be emergency projects, projects that have missed the start date and have a higher chance of liquidated damages; also projects that are deemed special and need to be done sooner due to political pressure).*

The Project Managers also provided information for thirteen highway construction projects. Seven of the projects were for the Florida Department of Transportation (FDOT), five for the Alabama Department of Transportation (ALDOT), and one for a city in Florida. Each project has its unique aspects and levels of complexity. Table 1 provides brief information of the projects included in this case study:

Table 1. *Projects Data from Company X*

ID	Description	Amount	Scope
P1	ALDOT IM-IMD-I020 (325) I-20 Birmingham AL	$38,557,835.61	Maintenance of traffic, roadway milling and crown corrections, resurfacing, borrow material, performance sod, guardrail replacement, interstate detours, unclassified excavation, concrete removal, bridge joint relocation and repair, bridge rail retrofit, installation of median barrier glare screen, removal and installation of bridge end slabs, cleaning and sealing concrete pavement, concrete pavement installation, grinding concrete pavement, pavement marking, sign installation, and overhead roadway sign structures
P2	ALDOT IM-STPAAF-I059 (338) Interchange Improvements I-59 at SR-216, Tuscaloosa County	$861,341.06	Maintenance of traffic, turn lanes and widening, grade, drain, base, and pavement
P3	ALDOT HPP-A146 (901) Safety Improvements on SR-5 at the Industrial Park Road in Woodstock, AL and at CR-24 in West Blocton, AL, Bibb County	$1,114,113.64	Maintenance of traffic , clearing & grubbing, borrow, concrete removal, borrow excavation, storm drainage, foundation backfill, base, grading, planning, resurfacing, pavement widening, pavement, and bridge culvert extension
P4	AL IM-NHF-I020 (327) I-20 Oxford, AL	$25,920,623.50	Maintenance of traffic, clearing and grubbing, removal of old bridge, removal of old drainage system, unclassified excavation, borrow excavation, structure, excavation, foundation backfill, roadbed stabilization, roadway milling, resurfacing, bridge widening, installation of new storm drainage system, concrete median safety wall installation, guardrail replacement, pavement markings, sign installation, and video detection system
P5	ALDOT Brewton City Streets Resurface Project ID: CDG-060918203 Contract: TFI-681-034998-920	$586,520.86	Maintenance of traffic, roadway milling, resurfacing, pavement markings
P6	FDOT Contract 11-011 City of Pensacola Street Rehabilitation	$1,289,223.00	Maintenance of traffic, roadway milling, resurfacing, concrete curb installation, manhole adjustments, and restriping
P7	FDOT Project ID: 422142-2-72-14 Contract: E3J53 Santa Rosa County I-10 Blackwater River Contract	$394,608.50	Maintenance of traffic, roadway milling, resurfacing, performance sod, and restriping of the asphalt roadway
P8	FDOT Project ID: 428657-1-52-01 – 428657-2-52-01 Contract: E3I84 National Sea Shore	$836,900.00	Maintenance of traffic , asphalt excavation and replacement on shoulder / roadway
P9	FDOT Project ID: 424106 & 419301-1-51-01 Contract: T3348 Escambia County SR-727 Fairfield Drive	$2,121,025.45	Maintenance of traffic, temporary barrier wall, clearing and grubbing, concrete pavement removal, regular excavation, borrow excavation, lime rock base, turnout construction, roadway milling, resurfacing, storm drainage work (pipe culvert, inlet concrete boxes, inlet tops, manholes, and valve boxes), concrete curb, concrete sidewalk, guardrail removal, guardrail installation, performance sod, pavement markings, sign installation, and loop assembly
P10	FDOT Project ID: 416936-1-52-01 Contract: E3H28 SR-85 Crestview, FL Okaloosa County	$933,092.33	Maintenance of traffic, clearing & grubbing, removal of existing concrete, roadway milling, resurfacing, manholes adjustment, valve boxes adjustment, drainage work (pipe culvert, and inlet concrete box), concrete curb, concrete sidewalk, performance sod, pavement markings, sign installation, and loop assembly
P11	FDOT Bayou Blvd Project ID: 416914-1-52-01 Contract: E3H68 Escambia County	$322,588.52	Maintenance of traffic, borrow excavation, roadway milling, resurfacing, manholes adjustment, valve boxes adjustment, performance sod, pavement markings, sign installation, and loop assembly
P12	Contract: 58-023 Valparaiso, FL	$574,017.02	Maintenance of traffic, roadway milling, resurfacing, regular excavation, borrow excavation, lime rock base, pipe culvert, concrete headwall, manholes adjustment, valve boxes adjustment, and pavement markings
P13	FDOT Project ID: 413440-1-52-01	$3,127,649.71	Maintenance of traffic, clearing and grubbing, removal of existing concrete, regular excavation, borrow excavation, roadway milling, resurfacing, manholes adjustment, valve boxes adjustment, concrete curb, concrete sidewalk, performance sod, pavement markings, sign installation, and loop assembly

Step – 0: Creating the Decision Matrix

Using the information provided by the Project Manager, the decision matrix was created including quantitative criteria C1 through C9 (numerical values) and qualitative criteria C10 through C13 for the evaluation shown in Table 2.

Table 2. *Case Study Decision Matrix*

Projects	Budgeted Cost	Bonus (Early Completion / day)	Liquidated Damages / day	Contract Days	Crews Needed	Pieces of Equipment	Number of Subs	Asphalt TNs	Project Distance (Miles)	Complexity	Cost Risk	Environmental Risk	Priority Due to Special Circumstances
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
P1	$38,557,835.61	$50,000.00	$50,000.00	270	20	15	22	212,636.00	5	VH	VH	ML	H
P2	$861,341.06	$2,500.00	$5,000.00	130	3	6	4	45,500.00	25	ML	VL	L	VL
P3	$1,095,443.00	$1,500.00	$3,500.00	90	3	6	5	8,127.00	15	L	H	H	VL
P4	$25,920,623.50	$25,000.00	$25,000.00	425	15	20	15	177,534.00	30	VH	VH	H	L
P5	$585,520.86	$750.00	$1,500.00	60	3	5	3	4,727.00	50	VL	VL	VL	VL
P6	$1,289,223.00	$1,500.00	$2,500.00	120	3	5	3	16,825.00	10	VL	L	VL	ML
P7	$394,608.50	$2,500.00	$5,000.00	30	3	5	2	4,026.00	20	VL	L	L	M
P8	$836,900.00	$3,500.00	$4,500.00	50	3	6	1	1,750.00	15	L	M	VHJ	VH
P9	$2,121,025.45	$3,500.00	$4,500.00	430	4	7	7	6,396.00	23	MH	H	ML	VL
P10	$933,092.33	$2,500.00	$5,500.00	127	4	10	4	6,623.00	15	M	L	L	L
P11	$322,588.52	$1,500.00	$2,500.00	60	3	6	3	2,616.00	5	VL	VL	VL	VL
P12	$574,017.02	$1,500.00	$2,500.00	60	3	5	2	5,820.00	50	VL	L	L	L
P13	$3,127,649.71	$2,500.00	$5,500.00	240	4	8	3	8,048.00	13	MH	MH	ML	ML

Step – 1 –Normalizing Quantitative Data

Each of the criterion value for each of the projects must be normalized. The following is an example of normalizing the Budgeted Cost for P2. pc_{ij} corresponds to the Budgeted Cost which in this case is $861,3541.06, *min ($pc_{ij}$)* corresponds to the minimum Budgeted Cost of all the projects in the list in this case $322,588.52 (for P11), *max (pc_{ij})* corresponds to the maximum Budgeted Cost of all the projects in the list in this case $38,557,835.61 (for P1). Using the formulation presented in step 1 of the methodology the results will be the following:

$$r_j = \frac{pc_{ij}-\min(pc_{ij})}{\max(pc_{ij})-\min(pc_{ij})} \quad P_{21} = (861,341.06 - 322,588.52) / (38,557,835.61 - 322,588.52) = \mathbf{0.0141}$$

This calculation should be repeated for each of the remaining projects' criteria for all the projects; Table – 3 shows the results of the normalization calculations. Please note that all values will range between 0 – 1 in order to transform the values into linguistic values.

Table 3. *Quantitative Values Transformed into Range 0 – 1*

Projects	C1	C2	C3	C4	C5	C6	C7	C8	C9
P1	1.000	1.0000	1.0000	0.6000	1.0000	0.0667	1.0000	1.000	-
P2	0.0141	0.0355	0.0722	0.2500	-	0.0667	0.1429	0.2075	0.4444
P3	0.0202	0.0152	0.0412	0.1500	-	-	0.1905	0.0302	0.2222
P4	0.6695	0.4924	0.4845	0.9875	0.7059	1.0000	0.6667	0.8335	0.5556
P5	0.0069	-	-	0.0750	-	-	0.0952	0.0141	1.0000
P6	0.0253	0.0152	0.0206	0.2250	-	-	0.0952	0.0715	0.1111
P7	0.0019	0.0355	0.0722	-	-	-	0.0476	0.0108	0.3333
P8	0.0135	0.0558	0.0619	0.0500	-	0.0667	-	-	0.2222
P9	0.0470	0.0558	0.0619	1.0000	0.0588	0.1333	0.2857	0.0220	0.4000
P10	0.0160	0.0355	0.0825	0.2425	0.0588	0.3333	0.1429	0.0231	0.2222
P11	-	0.0152	0.0206	0.0750	-	0.0667	0.0952	0.0041	-
P12	0.0066	0.0152	0.0206	0.0750	-	-	0.0476	0.0193	1.0000
P13	0.0734	0.0355	0.0825	0.5250	0.0588	0.2000	0.0952	0.0299	0.1778

Step – 2 –Linguistic Values

After the normalization in Step – 1, each new value between 0 and 1 can be converted into linguistic values using Figure – 1.
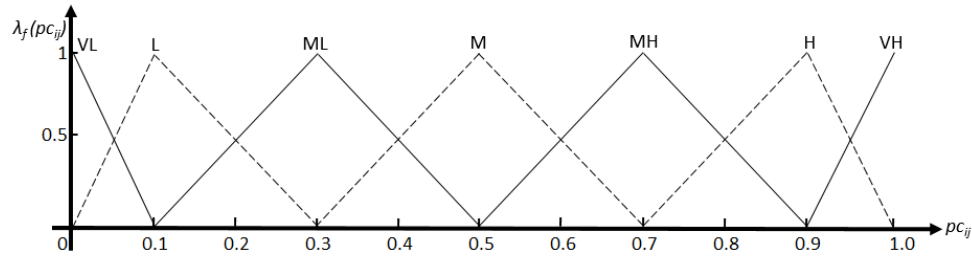

Figure – 1 Scale of Importance Chart for Crisp Performance Criteria Selection Values

The following is an example of changing the normalized value to a linguistic value for the Budgeted Cost for P2. pc_{ij} corresponds to the Budgeted Cost, which, in this case, is 0.0141. According to Figure – 1, the corresponding linguistic value will be *VL*.

This step should be repeated for each of the remaining projects' criteria for all the projects; Table – 4 shows the results of the calculations.

Table 4. *All Values Converted into Linguistic Values*

Projects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
P1	VH	VH	VH	MH	VH	MH	VH	VH	VL	VH	VH	ML	H
P2	VL	VL	VL	ML	VL	VL	L	ML	M	ML	VL	L	VL
P3	VL	VL	VL	L	VL	VL	ML	VL	ML	L	H	H	VL
P4	MH	M	M	VH	MH	VH	MH	H	M	VH	VH	H	L
P5	VL	VL	VL	L	VL	VL	L	VL	VH	VL	VL	VL	VL
P6	VL	VL	VL	ML	VL	VL	L	VL	L	VL	L	VL	ML
P7	VL	VL	VL	VL	VL	VL	VL	VL	ML	VL	L	L	M
P8	VL	VL	VL	VL	VL	VL	VL	VL	ML	L	M	VHJ	VH
P9	VL	VL	VL	VH	VL	L	ML	VL	ML	MH	H	ML	VL
P10	VL	VL	VL	L	VL	ML	L	VL	L	M	L	L	L
P11	VL	VL	VL	VL	VL	VL	L	VL	VL	VL	VL	VL	VL
P12	VL	VL	VL	VL	VL	VL	VL	VL	VH	VL	L	L	L
P13	VL	VL	VL	M	VL	L	VL	VL	L	MH	MH	ML	ML

Step – 3 –Linguistic Value to Triangular Fuzzy Value

The linguistic values are converted into triangular fuzzy values according to Table 5.

Table 5. *Linguistic Values & Correspondent Fuzzy Values to Evaluate Project Importance*

Scale	Abbreviation	Fuzzy Value
Very Low	VL	(0.0, 0.0, 0.1)
Low	L	(0.0, 0.1, 0.3)
Medium Low	ML	(0.1, 0.3, 0.5)
Medium	M	(0.3, 0.5, 0.7)
Medium High	MH	(0.5, 0.7, 0.9)
High	H	(0.7, 0.9, 1.0)
Very High	VH	(0.9, 1.0, 1.0)

The following is an example of changing the linguistic value to triangular fuzzy value for the Budgeted Cost for P2. pc_{ij} corresponds to the Budgeted Cost, which, in this case, is *VL*. According to Table – 5, VL will have a triangular fuzzy value of *(0.0, 0.0, 0.1)*.

This step should be repeated for each of the remaining projects' criteria for all the projects; Table – 6 shows the results of the calculations.

Table 6. *Triangular Fuzzy Values C1, C2, C12, C13*

Projects	C1			C2			C12			C13		
	α	β	γ	α	β	γ	α	β	γ	α	β	γ
P1	0.90	1.00	1.00	0.90	1.00	1.00	0.10	0.30	0.50	0.70	0.90	1.00
P2	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.10	0.30	0.00	0.00	0.10
P3	0.00	0.00	0.10	0.00	0.00	0.10	0.70	0.90	1.00	0.00	0.00	0.10
P4	0.50	0.70	0.90	0.30	0.50	0.70	0.70	0.90	1.00	0.00	0.10	0.30
P5	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10
P6	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.11	0.10	0.30	0.50
P7	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.10	0.30	0.30	0.50	0.70
P8	0.00	0.00	0.10	0.00	0.00	0.10	0.90	1.00	1.00	0.90	1.00	1.00
P9	0.00	0.00	0.10	0.00	0.00	0.10	0.10	0.30	0.50	0.00	0.00	0.10
P10	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.10	0.30	0.00	0.10	0.30
P11	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10
P12	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.10	0.30	0.00	0.10	0.30
P13	0.00	0.00	0.10	0.00	0.00	0.10	0.10	0.30	0.50	0.10	0.30	0.50

Step – 4 –Normalizing Qualitative Data

The fuzzy qualitative data in the decision matrix needs to be normalized by the use of a linear function procedure to preserve the range of each project's criteria. The final normalization value is represented by r_{ij}. The following is an example of normalizing the Budgeted Cost for P2. pc_{ij} corresponds to the Budgeted Cost, which, in this case, is *VL (0.0, 0.0, 0.1),* derived using the formulation presented in Step – 4 of the proposed framework. The results will be as follows:

$$r_{ij} = \left\{ \left(\frac{\alpha_{ij}}{\gamma_{ij}^{max}}, \frac{\beta_{ij}}{\gamma_{ij}^{max}}, \frac{\gamma_{ij}}{\gamma_{ij}^{max}} \right) \right\} \quad r_{21} = (0.00/1.00, 0.00/1.00, 0.10/1.00) = \mathbf{(0.00, 0.00, 0.10)}$$

This calculation should be repeated for each of the remaining projects' criteria for all the projects; Table – 7 shows the results of the qualitative normalization calculations.

Table 7. *Normalized Values before Weight Calculation C1 through C13*

Projects	C1			C2			C12			C13		
	α	β	γ	α	β	γ	α	β	γ	α	β	γ
P1	0.90	1.00	1.00	0.90	1.00	1.00	0.10	0.30	0.50	0.70	0.90	1.00
P2	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.10	0.30	0.00	0.00	0.10
P3	0.00	0.00	0.10	0.00	0.00	0.10	0.70	0.90	1.00	0.00	0.00	0.10
P4	0.50	0.70	0.90	0.30	0.50	0.70	0.70	0.90	1.00	0.00	0.10	0.30
P5	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10
P6	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.11	0.10	0.30	0.50
P7	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.10	0.30	0.30	0.50	0.70
P8	0.00	0.00	0.10	0.00	0.00	0.10	0.90	1.00	1.00	0.90	1.00	1.00
P9	0.00	0.00	0.10	0.00	0.00	0.10	0.10	0.30	0.50	0.00	0.00	0.10
P10	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.10	0.30	0.00	0.10	0.30
P11	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10
P12	0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.10	0.30	0.00	0.10	0.30
P13	0.00	0.00	0.10	0.00	0.00	0.10	0.10	0.30	0.50	0.10	0.30	0.50

Step – 5 –Objective Weight Calculation (Entropy Method)

Before starting the objective weight calculation, the fuzzy values need to be turned back into single values by using the defuzzyfication formula: $pc_{ij} = (\alpha_{ij} + 2\beta_{ij} + \gamma_{ij})/4$. The following is an example of calculating the objective weight of the Budgeted Cost for P2. pc_{ij} corresponds to the Budgeted Cost, which, in this case, is *VL (0.0, 0.0, 0.1)*, derived using the formulation presented in Step – 5 of the proposed framework. The results will be as follows:

Defuzzyfication $pc_{ij} = (\alpha_{ij} + 2\beta_{ij} + \gamma_{ij})/4 = (0.0 + (2*0.0) + 0.1)/4 = \textbf{0.0250}$

This calculation should be repeated for each of the remaining projects' criteria for all the projects; Table – 8 shows the results of the calculations.

Table 8. *Defuzzyfication Calculations C1 through C13*

Projects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
P1	0.9750	0.9750	0.9750	0.7000	0.9750	0.7000	0.9750	0.9750	0.0250	0.9750	0.9750	0.3000	0.8750
P2	0.0250	0.0250	0.0250	0.3000	0.0250	0.0250	0.1250	0.3000	0.5000	0.3000	0.0250	0.1250	0.0250
P3	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.3000	0.0250	0.3000	0.1250	0.8750	0.8750	0.0250
P4	0.7000	0.0250	0.5000	0.9750	0.7000	0.9750	0.7000	0.8750	0.5000	0.9750	0.9750	0.8750	0.1250
P5	0.0250	0.0250	0.0250	0.1250	0.0250	0.0250	0.1250	0.0250	0.9750	0.0250	0.0250	0.0250	0.0250
P6	0.0250	0.0250	0.0250	0.3000	0.0250	0.0250	0.1250	0.0250	0.1250	0.0250	0.1250	0.0275	0.3000
P7	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.3000	0.0250	0.1250	0.1250	0.5000
P8	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.3000	0.1250	0.5000	0.9750	0.9750
P9	0.0250	0.0250	0.0250	0.9750	0.0250	0.1250	0.3000	0.0250	0.3000	0.7000	0.8750	0.3000	0.0250
P10	0.0250	0.0250	0.0250	0.1250	0.0250	0.3000	0.1250	0.0250	0.1250	0.5000	0.1250	0.1250	0.1250
P11	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.1250	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250
P12	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250	0.9750	0.0250	0.1250	0.1250	0.1250
P13	0.0250	0.0250	0.0250	0.5000	0.0250	0.1250	0.0250	0.0250	0.1250	0.7000	0.7000	0.3000	0.3000

For the projection variable P_{ij} for each project in the matrix calculation, the following is an example used in calculating the projection variable P_{ij} of the Budgeted Cost for P2. pc_{ij} corresponds to the Budgeted Cost, which, in this case, is *0.0250*; the sum for column C1 is *1.9500*.

$$P_{ij} = {pc_{ij}}\Big/{\sum_{i=1}^{m} pc_{ij}} = 0.0250/1.9500 = \textbf{0.0128}$$

This calculation should be repeated for each of the remaining projects' criteria for all the projects; Table – 9 shows the results of the calculations.

Table 9. *Projection Variable Calculations C1 through C13*

Projects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
P1	0.5000	0.5571	0.5571	0.1697	0.5000	0.2887	0.3250	0.4063	0.0055	0.2155	0.1781	0.0714	0.2536
P2	0.0128	0.0143	0.0143	0.0727	0.0128	0.0103	0.0417	0.1250	0.1093	0.0663	0.0046	0.0297	0.0072
P3	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.1000	0.0104	0.0656	0.0276	0.1598	0.2082	0.0072
P4	0.3590	0.2857	0.2857	0.2364	0.3590	0.4021	0.2333	0.3646	0.1093	0.2155	0.1781	0.2082	0.0362
P5	0.0128	0.0143	0.0143	0.0303	0.0128	0.0103	0.0417	0.0104	0.2131	0.0055	0.0046	0.0059	0.0072
P6	0.0128	0.0143	0.0143	0.0727	0.0128	0.0103	0.0417	0.0104	0.0273	0.0055	0.0228	0.0065	0.0870
P7	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.0083	0.0104	0.0656	0.0055	0.0228	0.0297	0.1449
P8	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.0083	0.0104	0.0656	0.0276	0.0913	0.2320	0.2826
P9	0.0128	0.0143	0.0143	0.2364	0.0128	0.0515	0.1000	0.0104	0.0656	0.1547	0.1598	0.0714	0.0072
P10	0.0128	0.0143	0.0143	0.0303	0.0128	0.1237	0.0417	0.0104	0.0273	0.1105	0.0228	0.0297	0.0362
P11	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.0417	0.0104	0.0055	0.0055	0.0046	0.0059	0.0072
P12	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.0083	0.0104	0.2131	0.0055	0.0228	0.0297	0.0362
P13	0.0128	0.0143	0.0143	0.1212	0.0128	0.0515	0.0083	0.0104	0.0273	0.1547	0.1279	0.0714	0.0870

The entropy e_j for each criteria j is calculated using the following formula, where k is a perpetual and calculated through $k = 1/ln(m)$, where m represents the number of projects in the decision matrix. The following is an example of calculating the entropy e_j for criteria C1. P_{ij} corresponds to all the projects in C1 which are *P1(0.5000), P2(0.0128), P3(0.0128), P4(0.3590), P5(0.0128), P6(0.0128), P7(0.0128), P8(0.0128), P9(0.0128), P10(0.0128), P11(0.0128), P12(0.0128), P13(0.0128),* and *m = 13*. Using the formulation presented in Step – 5 of the framework, the proposed the results will be as follows:

$$e_j = -k \sum_j p_{ij} * \left(\ln(p_{ij})\right) \ or \ - \frac{\left[\sum_j p_{ij} * \left(\ln(p_{ij})\right)\right]}{\ln(m)}$$

$$= -\frac{[(P1 * \ln(P1)) + \cdots (P13 * \ln(P13))]}{\ln(13)}$$

$$= -\frac{[(0.5000 * \ln(0.5000)) + \cdots (0.0128 * \ln(0.0128))]}{\ln(13)} = \mathbf{0.5180}$$

This calculation should be repeated for each of the remaining projects' criteria; Table – 10 shows the results of the calculations.

Table – 10 *Objective Weights Calculations*

Projects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
P1	0.5000	0.5571	0.5571	0.1697	0.5000	0.2887	0.3250	0.4063	0.0055	0.2155	0.1781	0.0714	0.2536
P2	0.0128	0.0143	0.0143	0.0727	0.0128	0.0103	0.0417	0.1250	0.1093	0.0663	0.0046	0.0297	0.0072
P3	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.1000	0.0104	0.0656	0.0276	0.1598	0.2082	0.0072
P4	0.3590	0.2857	0.2857	0.2364	0.3590	0.4021	0.2333	0.3646	0.1093	0.2155	0.1781	0.2082	0.0362
P5	0.0128	0.0143	0.0143	0.0303	0.0128	0.0103	0.0417	0.0104	0.2131	0.0055	0.0046	0.0059	0.0072
P6	0.0128	0.0143	0.0143	0.0727	0.0128	0.0103	0.0417	0.0104	0.0273	0.0055	0.0228	0.0065	0.0870
P7	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.0083	0.0104	0.0656	0.0055	0.0228	0.0297	0.1449
P8	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.0083	0.0104	0.0656	0.0276	0.0913	0.2320	0.2826
P9	0.0128	0.0143	0.0143	0.2364	0.0128	0.0515	0.1000	0.0104	0.0656	0.1547	0.1598	0.0714	0.0072
P10	0.0128	0.0143	0.0143	0.0303	0.0128	0.1237	0.0417	0.0104	0.0273	0.1105	0.0228	0.0297	0.0362
P11	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.0417	0.0104	0.0055	0.0055	0.0046	0.0059	0.0072
P12	0.0128	0.0143	0.0143	0.0061	0.0128	0.0103	0.0083	0.0104	0.2131	0.0055	0.0228	0.0297	0.0362
P13	0.0128	0.0143	0.0143	0.1212	0.0128	0.0515	0.0083	0.0104	0.0273	0.1547	0.1279	0.0714	0.0870
e_j	0.5180	0.5269	0.5269	0.7745	0.5180	0.6497	0.7747	0.5728	0.8614	0.7813	0.8192	0.8070	0.7598
d_j	0.4820	0.4731	0.4731	0.2255	0.4820	0.3503	0.2253	0.4272	0.1386	0.2187	0.1808	0.1930	0.2402
Sum d_j							4.1097						
w_j	0.1173	0.1151	0.1151	0.0549	0.1173	0.0852	0.0548	0.1040	0.0337	0.0532	0.0440	0.0470	0.0584
Ling. Value	L	L	L	VL	L	VL	VL	L	VL	VL	VL	VL	VL
Triang. Fuzzy Value	(0, 0.10, 0.30)	(0, 0.10, 0.30)	(0, 0.10, 0.30)	(0, 0, 0.10)	(0, 0.10, 0.30)	(0, 0, 0.10)	(0, 0, 0.10)	(0, 0.10, 0.30)	(0, 0, 0.10)	(0, 0, 0.10)	(0, 0, 0.10)	(0, 0, 0.10)	(0, 0, 0.10)

The measurement of dispersion d_j is the intensity of each criterion, and the higher the number, the more important the weight of that criteria. The following is an example of calculating the dispersion d_j for criteria C1, and the entropy e_j is *0.5180*. Using the formulation presented in Step – 5 of the methodology, the results will be as follows:

$$d_j = 1 - e_j = 1 - 0.5180 = \mathbf{0.4820}$$

This calculation should be repeated for each of the remaining projects' criteria; Table – 10 shows the results of the calculations.

Finally, the weight w_j is determined for each criterion. The following is an example of calculating the weight w_j for criteria C1, where dispersion d_j is 0.4820 and the sum $\sum_{k=1}^{n} d_j$ is 4.1097. Using the formulation presented in Step – 5 of the proposed framework, the results will be as follows:

$$w_j = \frac{d_j}{\sum_{k=1}^{n} d_j} = \frac{0.4820}{4.1097} = \mathbf{0.1173}$$

This calculation should be repeated for each of the remaining project criterion and Table – 10 shows the results of the calculations. Using Figure – 1 and Table – 1, the objective weight needs to be converted back to triangular fuzzy value [w_j (α_j, β_j, γ_j)]. The following is an example of converting the weight result for C1 to triangular fuzzy value. According to Figure – 1, 0.1173 will be considered as the linguistic value of *L*, and according to Table – 1, the triangular value for *L* is *(0.0, 0.10, 0.30)*. This calculation should be repeated for each of the remaining projects' criteria; Table – 10 shows the results of the calculations.

Step – 6 –Objective Weight Normalization

The decision matrix is normalized v_{ij} using the objective weights. The following is an example of normalizing the Budgeted Cost for P2 in Table – 5. pc_{ij} corresponds to the Budgeted Cost, which, in this case, is *(0.0, 0.0, 0.10)*. Each pc_{ij} will be multiplied by the criterion's corresponding objective weight. For C1, the objective weight is (0.0, 0.10, 0.30). Using the formulation presented in Step – 6 of the methodology, the results will be as follows:

$$v_{ij} = w_j * pc_{ij} = (0.0 * 0.0)(0.10 * 0.0)(0.30 * 0.10) = (\mathbf{0.0, 0.0, 0.03})$$

This calculation should be repeated for each of the remaining projects' criteria; Table – 11 shows the results of the calculations.

Table 10 *Normalized Decision Matrix using Weight for C1, C2, C12, and C13*

Weights	0.00	0.10	0.30	0.00	0.10	0.30	0.00	0.00	0.10	0.00	0.00	0.10
		C1			C2			C12			C13	
Projects	α	β	γ	α	β	γ	α	β	γ	α	β	γ
P1	0.00	0.10	0.30	0.00	0.10	0.30	0.00	0.00	0.05	0.00	0.00	0.10
P2	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.01
P3	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.10	0.00	0.00	0.01
P4	0.00	0.70	0.27	0.00	0.05	0.21	0.70	0.00	0.10	0.00	0.00	0.03
P5	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.01	0.00	0.00	0.01
P6	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.05
P7	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.30	0.00	0.00	0.07
P8	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.10	0.00	0.00	0.10
P9	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.05	0.00	0.00	0.01
P10	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.03
P11	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.01	0.00	0.00	0.01
P12	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.03	0.00	0.00	0.03
P13	0.00	0.00	0.03	0.00	0.00	0.03	0.10	0.00	0.05	0.00	0.00	0.05

Step – 7 –Determining the Distance

Fuzzy TOPSIS ranks the job according to the distance between the PIS and the NIS; for this paper, the ideal solution for PIS and NIS have been determined as $PIS – A^+$ (1, 1, 1) and $NIS – A^-$ (0, 0, 0). The distance between two fuzzy values D_j can be determined for each project. The following is an example of determining the distance of the Budgeted Cost for P2 from Table – 9. pc_{ij} corresponds to the Budgeted Cost, which, in this case, is *(0.0, 0.0, 0.03)*. Using the formulation presented in Step – 7 of the methodology, the results will be as follows:

$d\left(pc_{ij}, pc_j^+\right)$, and $d\left(pc_{ij}, pc_j^-\right)$ are calculated with the formula for triangular fuzzy values as follows:

$$D_j^+ = \sum_{j=1}^{n} d\left(pc_{ij}, pc_j^+\right)$$

$$D_j^- = \sum_{j=1}^{n} d\left(pc_{ij}, pc_j^-\right)$$

$$d\left(pc_1, pc_2\right) = \sqrt[2]{\frac{1}{3}\left[(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2 + (\gamma_1 - \gamma_2)^2\right]}$$

$$d_j\left(pc_{21}, pc_{A+}\right) = \sqrt[2]{\frac{1}{3}\left[(0.00 - 1.00)^2 + (0.00 - 1.00)^2 + (0.03 - 1.00)^2\right]} = \mathbf{0.9901\ PIS}$$

$$d_j\ (pc_{21}, pc_{A-}) = \sqrt[2]{\frac{1}{3}\ [(0.00 - 0.00)^2 + (0.00 - 0.00)^2 + (0.03 - 0.00)^2]} = \mathbf{0.0173\ NIS}$$

This step should be repeated for each of the remaining projects' criteria for all the projects; Table – 12 shows the results of the calculations.

The D_j PIS and D_j NIS is the sum of the results of $d_{j\ PIS}$ and $d_{j\ NIS}$. The following is an example of determining the final distance D_j^+ & D_j^- of the Budgeted Cost for P2. The values of D_j^+ for C1 through C13 are: *0.9901, 0.9901, 0.9901, 0.9836, 0.9901, 0.966, 0.9901, 0.9422, 0.9772, 0.9836, 0.9966, 0.9901, and 0.9966.* The values of D_j^- for C1 through C13 are: *0.0173, 0.0173, 0.0173, 0.0289, 0.0173, 0.0058, 0.0173, 0.0883, 0.0404, 0.0289, 0.0058, 0.0173, and 0.0058.*

Using the formulation presented in Step – 7 of the methodology, the results will be as follows:

$$D_j^+ = \sum_{j=1}^{n} d\left(pc_{ij}, pc_j^+\right)$$

$$= 0.9901 + 0.9901 + 0.9901 + 0.9836 + 0.9901 + 0.966 + 0.9901 + 0.9422 + 0.9772 + 0.9836 + 0.9966 + 0.9901 + 0.9966 = \mathbf{12.8167}$$

$$D_j^- = \sum_{j=1}^{n} d\left(pc_{ij}, pc_j^-\right)$$

$$= 0.0173 + 0.0173 + 0.0173 + 0.0289 + 0.0173 + 0.0058 + 0.0173 + 0.0883 + 0.0404 + 0.0289 + 0.0058 + 0.0173 + 0.0058 = \mathbf{0.3077}$$

This step should be repeated for each of the remaining projects' criteria for all the projects; Table – 12 shows the results of the calculations.

Table 12. *Distances for PIS and NIS*

PIS	C1	C2	C12	C13	SUM	NIS	C1	C2	C12	C13	SUM
P1	0.8756	0.8756	0.9836	0.9678	12.1708	P1	0.1826	0.1826	0.0289	0.0577	1.2823
P2	0.9901	0.9901	0.9901	0.9966	12.8167	P2	0.0173	0.0173	0.0173	0.0058	0.3077
P3	0.9901	0.9901	0.9678	0.9966	12.8329	P3	0.0173	0.0173	0.0577	0.0058	0.2944
P4	0.8940	0.9177	0.9678	0.9901	12.2792	P4	0.1610	0.1246	0.0577	0.0173	1.1505
P5	0.9901	0.9901	0.9966	0.9966	12.8813	P5	0.0173	0.0173	0.0058	0.0058	0.078
P6	0.9901	0.9901	0.9963	0.9836	12.8771	P6	0.0173	0.0173	0.0064	0.0289	0.2142
P7	0.9901	0.9901	0.9901	0.9772	12.8776	P7	0.0173	0.0173	0.0173	0.0404	0.2136
P8	0.9901	0.9901	0.9678	0.9678	12.8265	P8	0.0173	0.0173	0.0577	0.0577	0.3060
P9	0.9901	0.9901	0.9836	0.9966	12.7940	P9	0.0173	0.0173	0.0289	0.0058	0.3637
P10	0.9901	0.9901	0.9901	0.9901	12.8513	P10	0.0173	0.0173	0.0173	0.0173	0.2598
P11	0.9901	0.9901	0.9966	0.9966	12.9167	P11	0.0173	0.0173	0.0058	0.0058	0.1443
P12	0.9901	0.9901	0.9901	0.9901	12.8747	P12	0.0173	0.0173	0.0173	0.0173	0.2194
P13	0.9901	0.9901	0.9836	0.9836	12.8131	P13	0.0173	0.0173	0.0289	0.0289	0.3291

Step – 8 –Determining the Nearness Value

The nearness value NV_i is the calculation that ranks the projects according to their criteria. The following is an example of determining the nearness value NV_i for P2, where $D_j^+ = 12.8167$ & $D_j^- = 0.3077$. Using the formulation presented in Step – 8 of the methodology, the results will be as follows:

$$NV_i = \left. D_j^- \middle/ (D_j^+ + D_j^-) \right. = \frac{0.3077}{12.8167 + 0.3077} = \mathbf{0.0234\ Which\ is\ ranked\ 5^{th}}$$

This step should be repeated for each of the remaining projects; Table – 13 shows the results of the calculations and respective ranks.

Table13. *Nearness Value Results and Ranking*

Projects	Dj+	Dj-	NVi	Rank
P1	12.1708	1.2823	0.0953	1
P2	12.8167	0.3077	0.0234	5
P3	12.8329	0.2944	0.0224	7
P4	12.2792	1.1505	0.0857	2
P5	12.8813	0.2078	0.0159	12
P6	12.8771	0.2142	0.0164	10
P7	12.8776	0.2136	0.0163	11
P8	12.8265	0.3060	0.0233	6
P9	12.7940	0.3637	0.0276	3
P10	12.8513	0.2598	0.0198	8
P11	12.9167	0.1443	0.0111	13
P12	12.8747	0.2194	0.0168	9
P13	12.8131	0.3291	0.0250	4

Conclusion

The complex decision-making process, considering both quantitative and qualitative elements, to schedule multiple construction projects would benefit from a more systematic approach as the one presented in the case study of this paper. The framework provides decision-makers with the ability to elect a construction schedule that encompasses all projects of a company, taking into account all constraints that govern each individual project. Depending on the size of the company and backlog amount, the framework would determine the most logical sequence based on the criteria of importance and with this, removing human uncertainty from the equation. The framework would also handle any changes occurring in the construction process with ease. The issue with changes in construction comes when an action is required to be performed by a person (decision-maker). The result will always be dependent on the ability of the person to make the decision. By using a systematic approach, such as the one used in this case study, the decision-making process can be easily produced and have more of an impact in managing construction projects. This is the ultimate goal that all managers and decision-makers are seeking. The companies in the construction industry are in it not only to produce a quality project, but also to make a profit. This means that more accurate decision-making is needed in order to reduce construction time, changes, and construction costs.

For this case study, the framework provided a ranking sequence that was based on the criteria used to evaluate project importance. The ranking is as follows (first to last): P1, P4, P9, P13, P2, P8, P3, P10, P12, P6, P7, P5, and P11. It can be observed that the resulting sequence is not dependent only on one criterion, but also on the criterion weight. For example, P2 is more important than P3 not because of the budgeted cost, but for the high asphalt tonnage required to complete the project. This result could help decision-makers make appropriate logistics to ensure that the plant has enough material available at the plant and job site, since this project is poised to start before P3. P8 is more important than P3, P6, and P10 not because of its budgeted cost, asphalt tonnage, complexity, crews, equipment, or bonuses, but for its high importance due to special circumstances. This gives the framework the ability to incorporate subjective criteria and also have an impact on the raking of the projects. Since the framework does take into account subjective criteria, decision-makers have more flexibility to achieve the best possible solution when scheduling multiple projects. The framework also eliminates any human uncertainty or doubt when large data is presented or used. The process has demonstrated the favorable sequence to take based on the nearness to the possible ideal solution, and yet, as far from the negative ideal solution. Each project was ranked appropriately based on its importance. The process or framework could be improved by providing adequate data for the selection criteria. This data will have more value or greater impact if the company has had the opportunity to collect the data from the very beginning of their construction operations. This will provide more accurate ranking because the data has gone through trial and error, which facilitates decision-makers to select the criteria that will have the best results when selecting the criteria for the project's importance evaluation. The process for this method has shown to be simple when used for scheduling multiple projects; however, the framework can be perfected by obtaining more data from other construction firms and comparing results. This will provide researchers with the opportunity to test the

framework and verify its implications in the construction industry. It is worth mentioning that the criteria can be very detailed if the information is available. The number of criteria is not bonded to thirteen; the company can choose the number of criteria to include and the kind of detailed information they want to evaluate. This could include crew performance, equipment production rates, project manager's capabilities, and other determining factors that affect the life cycle of a construction project. Thus, the framework might prove to be useful in other firms or areas in construction that require ranking in order to make better decisions when evaluating projects.

Finally, this method allows making changes to the schedule with ease. Some of the changes could arise as new projects are added to the backlog, or projects are completed. Some changes could arise from changed orders, bad weather, delivery of material, availability of labor/equipment, and unforeseen events. When these changes happen, modifications need to be done to the schedule and new rakings are needed to obtain the ultimate schedule. In the construction industry, changes occur frequently and cannot be ignored; therefore, the model needs to have the flexibility to incorporate design/field changes as they arise. Apart from giving an ultimate schedule alternative ranking, providing the ability to incorporate changes is a major benefit to the decision-making group.

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