Stochastic Framework for Cash Flow Forecasting Considering Owner's Delay in Payment by Use of Monte Carlo Simulation

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Financial supply is the most critical challenge contractors always confront in all construction stages. This issue is more vital in developing countries where rapid infrastructures development is necessary and public fund is scarce. Numerous models have been developed for forecasting cash flow, but the effect of late payments risk has not been considered in any of them. This paper aims to develop a new stochastic simulation-based framework for forecasting construction projects cash flow at the bidding stage, considering the effect of delay in payments. The time between carrying out an activity and receiving its corresponding payment has been modeled using a beta-distributed random variable based on experts judgment. Then, the Monte Carlo simulation is performed to extract the cumulative probability distributions of maximum required finance and financing cost. The model is applied to a highway project in Iran in which the contractor had foreseen a negative cash flow of up to 16% of the contract price. The results of applying the proposed model in which the payments lag time are simulated show that this figure is 42% and 25% for average and optimistic conditions respectively leading to a financing cost of 3.7% and 2.2% of the project budget.

Key words: Cash flow management, Late payment, Negative cash flow, Cost control**.**

Introduction

Cash flow is famously quoted by Lord Denning to be the life blood of construction industry. As pointed out by Navon (1996) the principal cause of most failure cases in construction companies is the lack of liquidity for supporting their daily activities rather than inappropriate management of other resources. A crucial factor for contractors to run a profitable business is the ability to procure adequate cash to execute construction operations with minimal financial expenses (Elazouni et al. 2004). The main factor affecting cash flow in a construction project is the time lag between contractor's expenses and payment collection. Financial stress may result from a combination of delayed payments with inaccurate cash forecasts and deficiencies in the cash flow management (Paul et al. 2012). Maeda et al. (2007) categorized delayed payment from clients as type of financial risk involving a high level of uncertainty. There seems to be a general agreement among contractors and public agencies that monthly payments are not made on time by public agencies, even though the timing of payments is explicitly laid down in standard agreement forms and general provisions (Abdul-Rahman et al. 2013). Therefore, it will be necessary for contractors to predict the financial risk of delay in client's payments in the bidding stage and consider the financing cost due to negative cash flow during the project execution in their bid. This research aims to develop a stochastic model for forecasting maximum finance and financing cost required by contractor to run the project continuously using Monte-Carlo simulation.

Cash flow Prediction of Construction Projects

Cash Flow Prediction Background

Cash flow, as accepted by most of the contractors and simulated in recent researches, is described as the cash in (net receipt) or cash out (net disbursement) resulting from receipts and disbursements occurring in the same interest period. Cash in is derived mainly from funds received in the form of billings (less retentions), release of retention funds, claims and change orders. Progress payments incur a contractually permissible delay after billing. Cash out, on the other hand is related to funds expended on a contract to pay bid costs, preconstruction costs (engineering, design, mobilization, etc.), materials and supplies, equipment and equipment rentals, payments of subcontracts, labor wages and overhead (Park et al. 2005;Al-Joburi et al. 2012). In a condition when flow of money into a project confronts delays, the net cash flow becomes negative (Paul et al. 2012). Unless payments are made in advance, many construction projects have negative net cash flow until the very end of the project when the final payment is received (Park et al. 2005). A Proper cash flow management, plays a strategic role even when a firm is not facing financial stress (Yang et al. 2013). In an attempt to reach a reliable level of liquidity, to run a sustainable business and to model the real business environment in the construction industry, it is essential to develop a financially sound analysis of the expected cash flow during all project stages (Han et al. 2014). Therefore, numerous cash flow forecasting techniques have been proposed in the field of cash flow management, which mainly differ in the method of integrating time and cost, accuracy and detail.

One of the earliest models for cash flow forecasting in the early planning stage of a project was the model developed by Reinschmidt et al. (1976) which integrated schedule and cost items. Furthermore, Ashley et al. (1977) tried to suggest a realistic model for cash flow prediction by subdividing the direct cost to a number of categories such as labor, materials, and equipment and assigning a fixed percentage of total cost to each one. The percentage assumed to be fixed during project execution time period. The most common and frequently used method to forecast cash flow of a project along with its progress, is extracting cash flow prediction based on project's activities schedule, assuming fixed variables such as activities duration, advanced payment amount, retention conditions, owner's payments frequency, etc. (Chen et al. 2005).The origin of existing models goes back to the methods developed by Rasdorf et al. (1991) under the assumption that cash flow is a function of the project schedule. Kenley (2003) thoroughly reviewed existing approaches for cash flow prediction based on varied advanced tools such as algebraic formulations, regression models, artificial neural network, fuzzy theory, etc., which none of them consider the time lag between the expenditure and receiving its proportionate payment.

The differences between cash outflows and project expenses as consequences of time lags in payment are highlighted in a work conducted by Park et al. (2005). Principal factors affecting cash flows can be categorized as deterministic and stochastic. The first category includes factors which can deterministically be known in the bidding stage, such as retention conditions, credit arrangement with suppliers or vendors, equipment rentals, etc. The second category consists of factors which are uncertain and should be dealt with stochastically such as duration of the project, times for receiving payments from the client, etc. Disquisition of the literature indicates that among the stochastic factors, effect of nondeterministic nature of project duration on cash flow has been addressed in the literature numerously but none of the existing models considers the delay in client's payments as a stochastic variable. Also, its effect on financing cost and maximum required finance has not been investigated. This research aims to develop a model for cash flow prediction considering stochastic nature of client payments.

Delay in client Payments

Variation in cash flow due to late payment can dramatically affect daily operations of the projects. Particularly in small businesses, it can be more minatory depending on the extent and duration of the delay. Delay in receipts from the owner can affect company's cash flow to the extent of leading to insolvency. (Abdul-Rahman et al. 2013). This issue originates from the fact that workers have to be paid whether or not the contractor has been paid (Low et al. 2003;Abdul-Rahman et al. 2013). Borrowing additional capital to fund cost overruns and any other financing methods leads to increase in financing cost incurred by contractors (Abdul-Rahman et al. 2013). Therefore it is crucial for contractors to assess the capability of the owners to procure fund timely in the bidding stage, based on previous performance of the owner. It means that a period with a negative cash flow must be forecasted and the financing cost required to procure enough liquidity for supporting daily activities should be included in the bidding price. It is expressed by Smith et al. (1999) that contractors may inflate the tender price if the clients have a record of late payment and the markups are affected by the adequacy of the clients project financing and their ability to pay on time. Searching the existing literature indicates that limited researches are carried out in the context of prediction delay in owner payments and its consequences during bidding stage.

Disbursement-Receipt Lag Time

In the existing models for estimating cash flow of projects, the time spent between performing an activity and receiving its proportionate payment by the contractor is called payment lag time, which is assumed to be constant in accordance with contractual payment conditions in most of the existing models. Like invoices which usually have a monthly frequency, owner payments are assumed to comply with the same rule and have a monthly period. However in many cases it takes more time from carrying out an activity to receiving its corresponding payment and it is practically demonstrated that it noticeably differs from client to client. To extend the cash flow prediction to a stochastic case, the underlying distribution of payments lag time should be extracted. This can be accomplished in two ways. Firstly, when historical data from similar clients and similar projects are available, the first alternative is to fit theoretical distributions to the available data and verify goodness-of-fit statistically. But when there is not enough historical data or obtaining and analyzing the required data takes long time with high costs, the second alternative is to extract the distributions based on experts judgment. The three point estimates method is familiar to most estimators, which takes the minimum, most likely, and maximum values to derive the distributions of simulation components. Among the existing common distributions, continuous beta is customarily assumed to be the underlying distribution of payment lag time variable since it can model asymmetrical situations and is always confined within definite limits (Schexnayder et al. 2005). The probability density function (PDF) of a general beta distribution is

$$f(x;\alpha,\beta,L,U)=\frac{(x-L)^{\alpha-1}(U-x)^{\beta-1}}{B(\alpha,\beta)(U-L)^{\alpha+\beta-1}} \qquad L\le x\le U,\alpha>0,\beta>0 \tag{1}$$

Where α and β are shape parameters, L is the lower bound, U defines the upper bound and $B(\alpha,\beta)$ is beta function, declared by

$$B(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}=\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt \tag{2}$$

So, to establish a beta distribution there are four parameters that should be defined α, β, lower and upper bounds. The last two parameters can be specified with three-point estimates, while additional pieces of information are needed to determine two other factors. As mentioned by (AbouRizk et al. 1991), any pair of the mean, variance, and mode can be employed to compute the shape parameters α and β. Knowing mean and variance of the stochastic variable, the analytic formulas for estimating the shape parameters are

$$\beta=\frac{\tau(1-\tau)^2}{\nu}+\tau-1 \tag{3}$$

$$\alpha=\frac{\beta\tau}{1-\tau} \tag{4}$$

Where τ and ν are defined as

$$\tau=\frac{\hat{\mu}-L}{U-L} \tag{5}$$

$$\nu=\frac{\hat{\sigma}^2}{(U-L)^2} \tag{6}$$

The same as Project Evaluation and Review Technique (PERT), the first two moments of the beta distribution can be estimated by

$$\hat{\mu}=\frac{L+4m+U}{6} \tag{7}$$

$$\hat{\sigma}^2=\frac{(L-U)^2}{36} \tag{8}$$

In equations 5-8 $\hat{\mu}$ is the mean, $\hat{\sigma}^2$ is the variance, and m is the most likely value.

Research Statement and Methodology

Having a clear prediction about client payment times will noticeably assist contractors to manage financial problems during project execution. Also it leads them not to bid for those projects that require finance exceeding company's maximum bank credit. Particularly, this issue becomes more significant in projects funded by clients facing financial crisis and cash shortage problem. The aim of this paper is to develop a stochastic framework for prediction of maximum finance required by contractor to support daily expenses of the project, and financing cost imposed to contractors caused by delays in receipts from client. The proposed framework is based on the financial terminology used by Au et al. (1986) with the difference that in the main model the payments lag time is assumed to be equal to zero which implies that the payment will be received from the client immediately after submitting an invoice. In the suggested model, a beta-distributed random variable $\mathbf{t_b}$ defining the payments lag time is added to the previous model. The payments lag time distribution is established using expert's judgment based on Eq. 3-8. Then Monte Carlo Simulation (MCS) is performed to extract the required parameters for different probability levels. Furthermore, the model is applied to a highway project in Iran, funded by the Construction and Development of Transportation Infrastructures Company (CDTIC).

CDTIC is an Iranian governmental body where all of the governmental funded transportation infrastructure projects are directed from the definition stage to their closure. All of engineering, procurement and construction works are put to tender in accordance with Iranian constitution. However since Iran's government is facing financial crisis and therefore liquidity issues, CDTIC also faces cash shortage problems causing numerous projects funded by this company to be suspended due to delay in payments. To extract the distribution of payments lag time in the projects directed by CDTIC, it is required to collect and analyze the data of projects completed in last five years. This is difficult and in some cases almost impossible to do for several reasons. The main cause is disinclination of employees to share financial information, since it seems as extremely confidential. Further, during data gathering phase, it is revealed that the company has not any particular format for organizing and archiving financial data. Each project manager in the firm is using arbitrary format with a different level of details for recording financial data. Therefore, the lack of organized financial data propelled the author towards utilizing experts judgment instead of using previous data.

Survey Conduction

To gather financial data required for fitting a distribution to the payments lag time variable, a questionnaire was developed and 35 interviews were conducted with CDTIC employees. Most of respondents were selected from project managers and their assistances, who directly manage the ongoing projects and are aware of all their financial characteristics. The cost of projects under supervision of candidate interviewees sums up almost 30% of the firm's total projects. The demographic profiles of the experts are shown in Table 1. The interviewees were asked to determine the minimum, most likely and maximum duration for the payments lag time based on their experiences over three periods of the project progress. According to pilot study it is guessed that client payment routine differs over different progress levels of the project. Therefore the project execution life cycle is divided to three stages with of 0-30%, 30%-70% and 70%-100% progress. Also, it is surmised that payments lag time distribution in last three years may differ from the years before due to the financial crisis. So, the respondents were requested to fill the questionnaire once based on projects that have been carried out over last three years, and again based on projects executed four years ago and before.

Table 9-Brief Summary of Respondents' Characteristics

General Information	Frequency of Respondents	Percentage of Respondents
Job Title		
Project Manager	19	54.3
Project Manager's Assistant	11	31.4
Financial Manager	5	14.3
Work Experience (Years)		
Fewer than 6	2	5.7
6-10	4	11.4
10-14	7	20.0
More than 14	22	62.9

Table 10- Final Result of surveys for Extracting Lag Time Distribution

Project Execution Period	Billing Time (Month)	Project Progress Level		
		0% - 30%	30% - 70%	70% - 100%
Last 3 years	Minimum	0.8	0.5	0.5
	Most Likely	2.4	3.4	2.1
	Maximum	10.4	11.5	5.8
Before	Minimum	0.7	0.5	0.5
	Most Likely	2	2.3	1.9
	Maximum	5.9	5.7	3.2

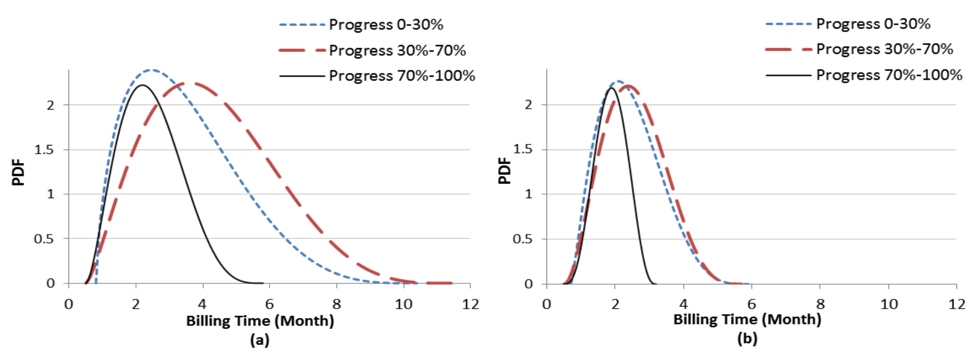

Figure 10 - Payments lag time beta distribution, fitted based on expert's judgment.
(a) Project executed in last 3 years (b) Projects executed before 3 years ago

The beta distribution for payment lag time was extracted based on the average of questionnaires' results and equations 3-8 (See Table 2). As shown in Fig. 1 the payment lag time distribution in projects conducted by CDTIC in the last three years tends to the right side of the chart, which means it takes more time for owners to procure enough liquidity for the projects. Also it is revealed that in the middle stages of the project it becomes more difficult for the client to pay the invoices. The extracted distribution is not unique for all situations and this distribution is strongly dependent on the client behavior and could vary from client to client.

Cash Flow Model Formulation

The cash flow model is developed from the contractors' perspective as shown in Fig. 2. The equations in this section are presented conforming to the financial terminology used by Au and Hendrickson (1986) with a few modifications to include the effect of delay in client payments.

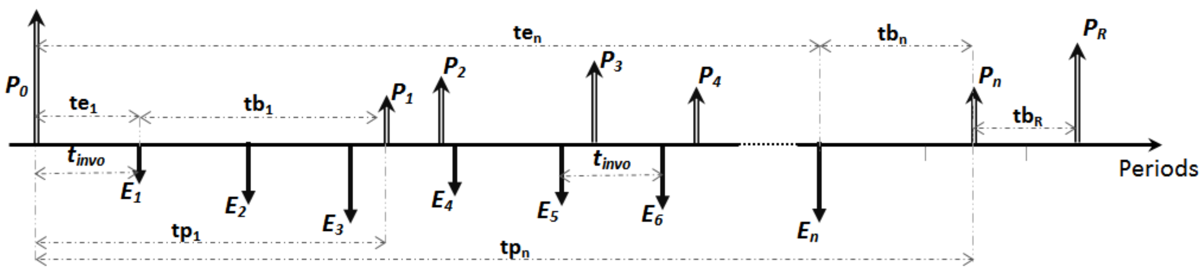

Figure 11 - Disbursement-Receipt profile for a typical project

The contractor submits payment requests (S_i) regularly at the end of each period based on project outflow in the corresponding period and receives its proportional payment (cash inflow).

$$S_i = \mathrm{K}\,.\,E_i \tag{9}$$
$$P_i = S_i - c_r.\,S_i - \frac{S_i}{Cost_t}.\,P_0 \tag{10}$$

Where K is a multiplier (K>1) to apply the fee percentage to a given amount of cash outflow in cost-plus fee contracts. E_i = project cash outflow during a typical project period i, which encompasses costs of overheads and taxes in addition to the direct costs including the costs of materials, equipment, labor and subcontractors.

P_i =Received payments (cash inflow), c_r = retention percentage, $Cost_t$ = estimated total cost of the project and P_0 =advance payment. The second term of Eq. 10 represent retention money which is subtracted from all of the invoices (S_i) and will be restored after project's completion (P_R) and the third term of Eq. is for advance payment depreciation.

$$te_i = i * t_{invo} \tag{11}$$
$$tp_i = te_i + tb_i \tag{12}$$
$$t = \{tp_1, tp_2, \dots, tp_n\} \cup \{te_1, te_2, \dots, te_n\} \qquad (j = 1{:}\,2n) \tag{13}$$

The disbursements are assumed to be exerted at times te_i and the receptions will be at times tp_i as defined in Eq. 11-12. In calculation of payment times, a random variable with beta distribution (tb_i) is utilized to model the delay in payments based on the distribution extracted from experts judgment as depicted in Fig. 1 (a).

$$D_{t_j} = \sum E_i \qquad \{i \mid te_i \le t_j\} \tag{14}$$
$$R_{t_j} = \sum P_i \qquad \{i \mid tp_i \le t_j\} \tag{15}$$
$$F_{t_j} = R_{t_j} - D_{t_j} \tag{16}$$
$$E_{t_j} = D_{t_j} - D_{t_{j-1}} \tag{17}$$

D_{t_j} = cumulative cash out incurred by contractor from beginning of the project to time t_j, R_{t_j}= cumulative receipt (cash in) from the client up to time t_j, F_{t_j} = cumulative net balance at time t_j which is calculated based on Eq. 16.

$$I_{t_j} = \begin{cases} \left((1+r)^{(t_j - t_{j-1})} - 1\right)\left(F_{t_{j-1}} + \frac{E_{t_j}}{2}\right), & F_{t_{j-1}} < 0 \\ \left((1+r)^{(t_j - t_{j-1})} - 1\right)\left(\frac{E_{t_j} - F_{t_{j-1}}}{2}\right), & F_{t_{j-1}} - E_{t_j} < 0 \\ 0, & F_{t_{j-1}} - E_{t_j} \geq 0 \end{cases} \tag{18}$$

Typically, cash procurement through the banks' credit lines or other financial sources incurs financing costs. The financing cost charged to the project at the end of time t_j is I_{t_j} which is calculated using Eq. 18. A positive cumulative net balance at time t_{j-1} ($F_{t_{j-1}}$) implies that the contractor debit is null and the contractor can use the surplus cash to finance activities during the current period. If the surplus cash can completely cover the amount of E_{t_j}, the contractor borrows no cash and the last line of Eq. 18 applies. Otherwise, the contractor will pay financing costs only for the amount of money he borrows in excess of the surplus cash as in second line of Eq. 18. In case of negative $F_{t_{j-1}}$, First line of Eq. 18 applies to calculate the financing cost. In the above equation r represents interest rate per month. It is assumed in the current cash flow model that the contractor will pay all the financing costs when the last payment is received. Therefore, the periodical financing costs are compounded by applying Eq. 19.

$$I'(t_j) = \sum_{j=1}^{2n} I(t_j)(1+r)^{t_{2n} - t_j} \tag{19}$$

Illustrative Example

To illustrate the new methodology presented, a simple case study has been conducted. The illustrative case is a highway with 150 km length which is funded by CDTIC. The project is put to tender in May-2012 and the project started 9 month later in March- 2013, the project duration is estimated 12 month and the project budget is almost US$120,000,000. According to the signed contract the invoice submission frequency is monthly and retention ratio is 10% of each payment which is retained after completion of construction. Advanced payment is 10% of total project cost which is paid at once in the beginning of the project.

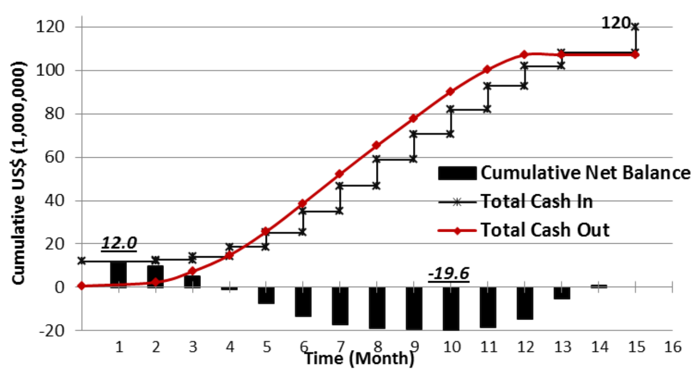

Figure 12 – Contractor's Estimate for Income-Expense Profile at the Bidding Stage

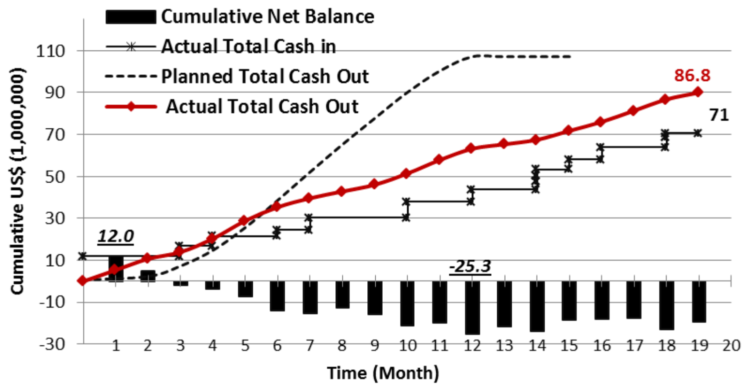

Figure 13 –Projects Actual Income-Expense Profile vs. Planned Profile

The project delivery system is design-build with a lump-sum price. In the bidding stage the contractor assumed that the maximum payment lag time is 1 month and the fee percentage considered to be 12%. The contractor preliminary forecast for the project income-expense profile is displayed in Fig. 3 which is established based on above assumption and the project schedule attached to the contract. At the time that this research is carried out, although the contractual project completion time has been passed, the project has not been completed and it has 81% physical progress. It means that the project execution is delayed. When the causes are investigated, it is revealed that the principal reason is client delay in payments due to financial issues CDTIC is confronting as discussed in previous sections. As shown in Fig 4. project actual progress rate was more than its planned value at the beginning of the project, but after the 4th month, when the delays in payments are emerged, the progress rate was declined and the net balance became negative. After the 6th month the contractor decreased the project execution speed to dominate the cash shortage problem and afterwards the planned and actual progress diverged with a high rate. This routine continued to the 14th month. The maximum value of client's debit occurred at the 12th month with a value of almost 25.3 million dollars. At month 14 the client tried to hasten in payments routine. Afterwards, the contractor tried to increase the progress rate but still there was a large gap between scheduled and actual progress. Therefore it is obvious that the owner delay in cash procurement caused the project to be extended up to double of its first estimated duration. On the other hand, according to what mentioned in survey conduction section, it was distinctly evident in the bidding stage that the mentioned client could not procure required cash timely due to financial problems and the delay in payment could be estimated by the contractor. The proposed framework has been implemented and the cash flow of described project has been simulated. Monte Carlo simulation has been performed by use of beta distribution displayed in Fig. 1 (a). The main variable for a Monte Carlo simulation is the number of iterations. Generally an iteration number equal to or greater than 1,000 is sufficient.

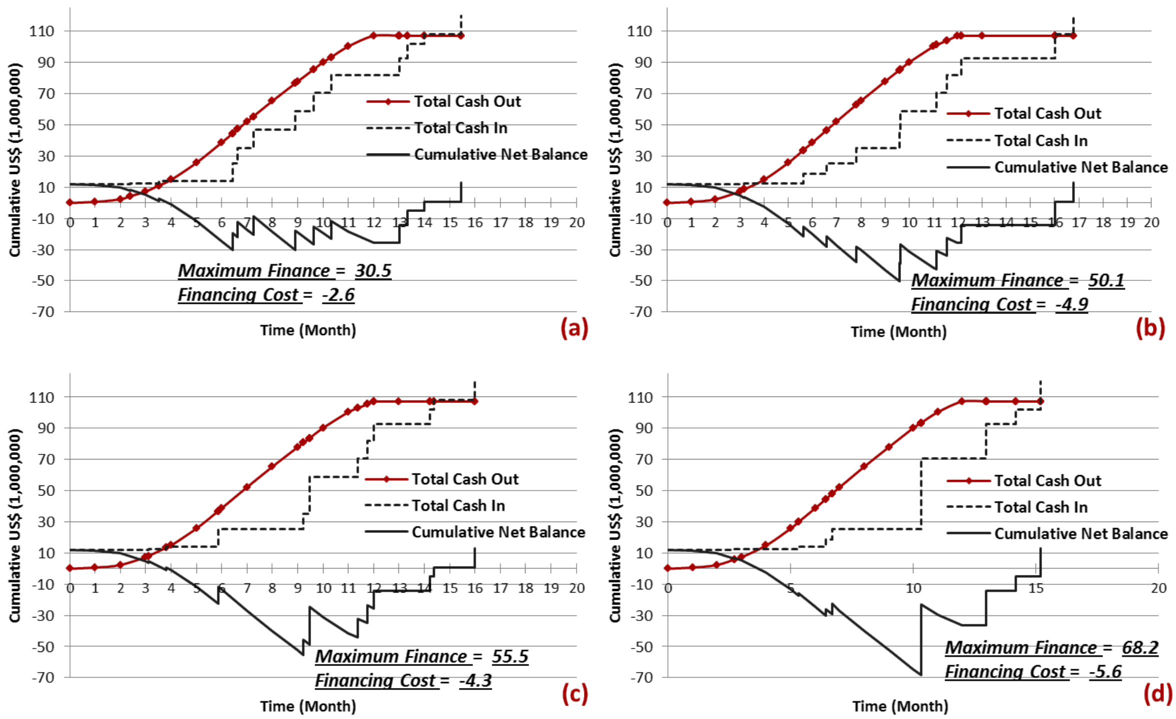

Figure 14 – Four Iterations Results of Monte Carlo Simulation for Cash Flow Forecasting

After formation of cash-in and cash-out profile in each iteration, maximum required finance and financing cost parameters are calculated. Fig. 5 displays 4 iterations results of Monte Carlo simulation. In each chart of Fig.5 the cash in, cash out and net balance variables are shown during project execution periods. The cash-in profile is built using Eq. 12 – 15 and mentioned beta-distributed random variable for payment lag time. The cash-out profile is extracted based on project schedule which is constant in all iterations. The net balance is also computed using Eq. 16.

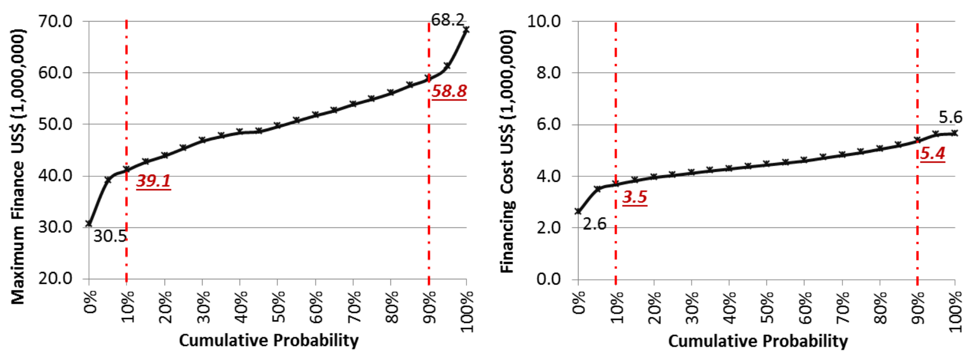

Figure 15 – Cumulative Probability Distributions of Maximum Required Finance and Financing Cost

The cash in profile is changed in each cycle as depicted in 4 cases of Fig. 5, which caused the maximum required finance to vary from 30.5 million dollars in chart (a) to 68.2 million dollars in chart (d). Also, the financing cost varies from 2.6 up to 5.6 million dollars. In the calculation of financing cost it is assumed that the interest rate per month is equal to 2%. The maximum required finance and financing cost cumulative probability distributions produced by proposed framework are presented in Fig. 6 (a) and (b) respectively. The results reveal that for completing the project at contractual duration, in optimistic conditions the maximum required finance equals to 30.5 million dollars and the financing cost that should be incurred by contractor are 2.6 million dollars, and for the pessimistic conditions these parameters are respectively 68.2 and 5.6 million dollars. The average values of maximum required finance and financing cost for all iterations are respectively 49.9 and 4.4 million dollars. In ideal circumstance as shown in Fig. 3 the maximum required finance was 19.6 million dollars which means the owners delay in payment causes the maximum required finance to be altered dramatically. Also, it imposes noticeable financing cost to contractors.

Conclusion

In this study, a new stochastic framework for forecasting cash flow of construction projects has been introduced. The proposed model is simulation-based and is developed for the purpose of estimating maximum required finance that should be procured by contractors to support daily expenses and financing cost incurred by them due to the risk of payments delay by owners. The beta-distributed random variable for lag time of each progress payments have been introduced based on expert's judgment. Then, the Monte Carlo simulation performed to extract the maximum required finance and financing cost cumulative probability distributions. The operational logic of the model and an example application to a highway project are included in the paper. The results of model's application show that it operates well and produces realistic results regarding to the uncertainty inherent in the payments lag time. However, this conclusion cannot be generalized and the proposed model should be tested in several projects for reliable evaluation. Further case studies are being carried out for this purpose; this paper comprises only the development of the model.

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