

# College Mathematical Problem Solving Abilities

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This paper reports the results of a diagnostic investigation concerning students' abilities to problem-solve while calculating area, converting inches to a decimal equivalency, calculating volume and volume including slope. Integral to this investigation is whether or not there is a difference in performance between students using paper-and-pencils or using calculators to problem solve. Subjects within this study included 172 upper level construction students. This study found that students generally have difficulty solving these type problems with an average score of 62.8%. Students that use a calculator are significantly better at providing an accurate solution than those using paper-and-pencil. In all cases investigated, as the problem became more difficult students were more likely to draw images in an effort to clearly model the problem variables and relationships.

**Key Words:** Mathematical problem-solving, Area, Conversion, Volume, Calculator, and Paper-and-pencil

## Rational

Activities directed toward teaching and evaluating a student's cognitive skills pervades almost all areas of instruction and is involved in most professional areas of learning. The mission of educational institutions is to impart knowledge and to teach cognitive skills, which includes skills in problem-solving ability (Frederikson, 1984). There is a significant body of empirical literature on cognition, problem solving, and mental modeling (Anderson, 1983; Johnson-Laird, 1983; Johnson-Laird et al., 1989; Schoenfeld, 1992).

The state of Texas established the College Readiness Program (HB1, 2006) which is a collaborative project between the Texas Education Agency and the Texas Higher Education Coordinating Board to develop minimum standards for curricula, professional development materials in English, math, science, and social studies. Within these college-readiness standards a graduating high school senior must possess and demonstrate proficiency in these selected curricular areas (TCRS, 2007). Section I numerical reasoning states that a student must be able to perform accurate computations with real numbers such as add, subtract, multiply, and divide and solve problems involving rational numbers, ratios, percents, and proportions in context of situation. Section IV measurement reasoning requires that a student must be able to convert within a single measurement system by converting between basic units of measurement within a system.

It has often been noticed that some construction management students cannot seem to be able to solve problems involving simple arithmetic. It appears to be getting worse while the secondary state educational system and University claims to be setting higher and higher entrance standards, program cut levels, and exit requirements. This study investigates mathematical problem solving through a diagnostic test given within a construction equipment course. This activity may at first seem out of place; however the first portion of the course is dedicated to

providing students with project take-off instruction targeting heavy-civil work. The instructional strategy is to provide students with project and task knowledge to be recalled and applied within later activities that require them to select appropriate project equipment, calculate machine and attachment production times, and forecast project equipment costing. The purpose of this research is to identify where students have difficulty solving simple math problems, and to provide the understanding necessary to develop a teaching activity that will affect a positive change in student performance.

Problem solving is a formal logic system that includes principles, which identify well-formed formulas and rules of derivation, which when correctly applied lead to correct solutions. Therefore, one way to assess a student's problem solving ability is to vary accompanying aspects of the task to identify the affect on the subject's performance (Kobrin & Young, 2003). Rouse and Morris (1986) argued that varying the task can be an important factor that influences the mental model of the problem set, the procedural algorithm invoked, and its resulting solution.

Within the literature there is a "math war" underway and it has continued now for twenty years. For over thirty years students have had classroom access to calculators and computers and this has had an affect upon the teaching strategies and learning within the mathematics curriculum (Barton, 2000; Hembree & Dessart, 1986). There are two sides to the controversy. The first is defined as "*fuzzy math*" or the reformers (Starr, 2002; Van de Walle, 1999) which is intended to allow students to discover broad mathematical concepts on their own and then reinforce them with repetition and practice. The "*traditional math*" or the basics is the old "kill and drill" which concentrates on computational skills using automatized paper-and-pencil, drill and practice (Van de Walle, 1999). The reformer argue that students have failed to retain the paper-and-pencil skills previously learned, that any tool that can enhance student performance should be implemented, and the reduction in activity time provides students with more problem-solving time with which to discover new concept relationships (Anthony, 1999; Koop, 1982). Traditionalists argue that technology should not be used to replace basic understanding and intuitions, students who do not understand basic skill may not have success in future classes, students will not learn computational algorithms which may be detrimental to the learning process and advanced development (McNamara, 1995). Both sides agree that mean performance is greater when students use calculators. It is clear that the students within the current equipment coursework are products of the math reformation.

Comprehending mathematical word problems correctly and then translating them into organized expressions and equations, is a crucial part of doing construction based management, science, and technology. While there are numerous investigations that have examined mathematical reasoning and problem solving with respect to disabilities (Rourke & Conway, 1997) and elementary education (Johnson, 1944), this researcher has not been able to identify any empirical research on arithmetic calculations at the university level that are investigated by this study. It might be said that our academic body of math educators have passed over higher education or in the words of Devlin (1999), a senior mathematics researcher at Stanford:

First, living in today's world does not require that everyone must have arithmetic ability. Second, even if it did, considerable evidence suggests that such an ability cannot be taught in school settings. ... Today, we have cheap and readily available machines that do

arithmetic. Today's citizen no more needs to know how to add, subtract, multiply, or perform long division than to be able to plow or ride a horse. (p. 3)

Clearly, a construction student must be able to calculate area, convert an inch measure into its decimal equivalency, calculate volume, and integrating the slope of a ditch slope into a volume measurement and that he or she will not always have a calculator on their person when the time comes to solve a problem integrating one or more of these unifying math concepts. This study will investigate the differences in paper-and-pencil and calculator problem solving. It is hypothesized that paper-and-pencil students will not be as successful as calculator students while computing their solutions.

### **Methodology**

The design of the study was a  $2 \times 2 \times 4$  factorial which included two between-subject factors (instrument type and instrument iteration). The instrument type had two levels (no-calculator and calculator) as did the instrument iteration (A and B). The mathematical problem-solving included (area, conversion, volume, and slope).

The sample ( $N = 172$ ) consisted of students enrolled in a construction equipment course. Two instrument type treatment populations were sampled as well as four instrument populations within each of those groups: the paper-and-pencil group in the spring and summer of 2005 ( $n = 80$ ) included both A ( $n = 41$ ) and B ( $n = 39$ ) instrument iterations, while the calculator group in the spring and summer of 2006 ( $n = 92$ ) also included iterations A ( $n = 47$ ) and B ( $n = 45$ ). All students had been accepted into the upper level of program study and included these university levels: U2 ( $n = 3$ ), U3 ( $n = 41$ ), and U4 ( $n = 128$ ). Ninety-one percent of the sample was male, with a mean age of 22.7, and an average final grade point of 82.3 percent. The subjects were given two separate quiz points for taking both the Test of Logical Thinking (TOLT) and the diagnostic test. Subjects were randomly assigned by student selected seating and by odd and even iteration handout. A review of the research was conducted and acceptance was provided by the Institutional Review Board for Human Subjects.

The TOLT is an instrument that is used to measure five modes of formal thinking: 1) controlling variables, 2) proportional reasoning, 3) combinatorial reasoning, 4) probabilistic reasoning, and 5) correlational reasoning. A subject's level of thinking is evaluated by not only the question solution but how they justify their solution (Tobin & Capie, 1981). The on-line TOLT is used to determine if between-instrument subjects have the same reasoning ability.

The diagnostic instrument developed required students to solve simple problems involving calculating square yard area, converting inches to a decimal equivalency, cubic yard volume, and the volume of a sloped ditch within a 10 minute time period. Only one question per concept was developed, in that, the mathematics was first introduced to the students in the fifth grade and academic practice can be demonstrated as continuing into the present. Two test iterations with differing length, width, and depth values controlled for cheating by alternating the test iteration to ever other student. The one hour course sections were scheduled between 9:10 AM and 1:40

PM and the tests were given on Wednesdays. The questions as well as the solution algorithms are provided in Appendix A.

### Analysis

Instrument scoring was conducted by evaluating for correctness of response. A correct response was assigned 1 point and an incorrect response was assigned 0 points. During the scoring of the data set, an additional and separate measure was established. That measure represented the presence of drawn images within the problem work area. This might provide insight into the graphical nature of student problem solving. Drawing an image was valued as 1 point and 0 points for no drawing.

In order to answer the questions raised in this study and to provide the appropriate statistical analysis, all data points were first fit to the Shapiro-Wilk test for an analysis of Normal Goodness-of-Fit and it was found that the data was not normally distributed. Therefore, two different nonparametric tests were used in order to support the constancy and reliability of the findings. The Wilcoxon/Kruskal-Wallis test and the Van der Warden test, both using a normal and a chi-square approximation ( $p \leq .05$ ), were performed by using JMP 5.1 (SAS Institute, 2003) to test differences among groups. These two tests almost always gave the same conclusions, and thus reinforced one another in terms of the reported results of this research study. Only the Wilcoxon/Kruskal-Wallis test results will be reported where both tests found significant differences.

An analysis of group differences on the independent factors of instrument type and the instrument iteration by the dependent factors of course final grade and the TOLT was conducted. This analysis was necessary to determine if the sample populations were the drawn from the same population. No significant differences among or between these measures were found. Therefore, the sampled populations are equally representative of the same population of subjects.

#### Total Percent Correct

Significant differences were found on the percent correct for problem solutions with an instrument mean of 35.0% ( $SD = 0.26$ ). The calculator group ( $M = 0.40$ ,  $SD = 0.23$ ) significantly scored higher ( $p = .0063$ ) than the paper-and-pencil group ( $M = 0.30$ ,  $SD = 0.28$ ). No significant differences were found on the number of problem drawings with mean of 25.7% ( $SD = 0.14$ ).

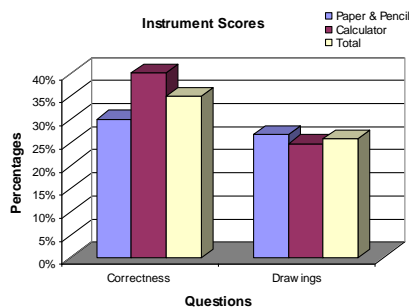


Figure 1: Subject performance on diagnostic instrument.

A “Bug” (VanLehn, 1990) analysis was conducted of the procedural algorithms recorded by the subjects while problem solving for each of the instrument questions. Two procedural errors were identified that accounted for 27.8% of the solution and problem solving errors. These errors were identified as a ‘solution round error’ and a ‘computational round error’. The solution round error was where the solution was not rounded as requested in the problem’s instructions and the computational round error was committed by using a rounded value during the solution computation. Both of these errors produced incorrect answers, but the algorithmic process that the subject used was found to be correct. If the solution round errors were allowed, the problem-solving mean would be increase to 43.5% ( $SD = 0.25$ ) and if the computational round errors were allowed, the problem-solving mean would further be increase to 62.8% ( $SD = 0.33$ ). By allowing this scoring adjustment, the calculator group ( $M = 0.76$ ,  $SD = 0.28$ ) continued to significantly score higher ( $p = <.0001$ ) than the no-calculator group ( $M = 0.47$ ,  $SD = 0.32$ ). Due to the possible confounding of differences in problem values between instrument iterations and the potential of the first and second questions queuing subject’s to commit the rounding errors within the third question, the remainder of the reported problem scores will take into account these errors by reporting performance after these errors have been allowed and scored as correct.

#### Percent Correct by Question

On question one which evaluated area calculations, the mean correctness score was 60.5% ( $SD = 0.49$ ). Significant differences were found for problem solutions between the calculator group mean ( $M = 0.67$ ,  $SD = 0.47$ ) and the paper-and-pencil group mean ( $M = 0.53$ ,  $SD = 0.50$ ). The calculator subjects scored significantly higher ( $p = .0472$ ) than did the paper-and-pencil subjects. No significant differences were found on the number of problem drawings which had a mean of 14.0% ( $SD = 0.35$ ), the paper-and-pencil group had a mean of 0.064,  $SD = 0.33$  and the calculator group had a mean of 0.076,  $SD = 0.36$ .

For question two which evaluated decimal equivalency conversion, the mean correct solution score was 85.5% ( $SD = 0.35$ ). Significant differences were found for problem solutions between the calculator group mean ( $M = 0.99$ ,  $SD = 0.10$ ) and the paper-and-pencil group mean ( $M = 0.70$ ,  $SD = 0.46$ ). The calculator subjects scored significantly higher ( $p = .0472$ ) than did the paper-and-pencil subjects. No subjects drew problem images while problem solving for this question.

For question three, calculating a volume in cubic yards, the mean correctness of solutions was 42.4% ( $SD = 0.50$ ). Significant differences were found for problem solutions between the calculator group mean ( $M = 0.63$ ,  $SD = 0.49$ ) and the paper-and-pencil group mean ( $M = 0.19$ ,  $SD = 0.39$ ). The calculator subjects scored significantly higher ( $p = <.0001$ ) than did the paper-and-pencil subjects. No significant differences were found on the number of problem drawings which had a grand mean of 6.4% ( $SD = 0.25$ ). The paper-and-pencil group’s mean was 0.04,  $SD = 0.28$ ) and the calculator group’s mean was 0.03,  $SD = 0.15$ ).

Question four which involved calculating a ditch volume with sloped sides, the mean correctness of solutions was 17.4% ( $SD = 0.38$ ). Significant differences were found for problem solutions between the paper-and-pencil group mean ( $M = 0.24$ ,  $SD = 0.43$ ) and the calculator group mean

( $\underline{M} = 0.12$ ,  $\underline{SD} = 0.33$ ). The paper-and-pencil subjects scored significantly higher ( $p = .0429$ ) than did the calculator subjects. Significant differences were not found on the number of problem drawings with a grand mean of 82.6% ( $\underline{SD} = 0.38$ ). The paper-and-pencil group's mean was 0.44,  $\underline{SD} = 0.35$ ) and the calculator group's mean was 0.38,  $\underline{SD} = 0.42$ ).

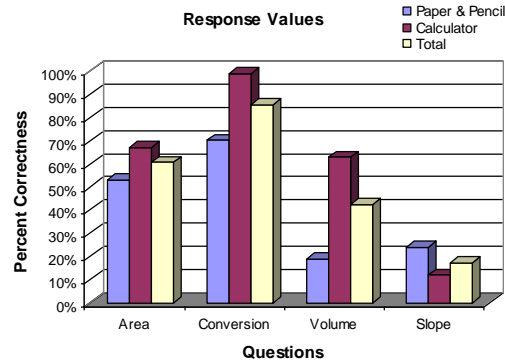


Figure 2: Student performance on correctness by question.

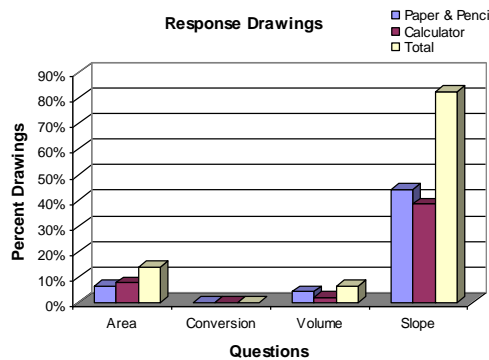


Figure 3: Student drawing diagram by question.

### Algorithmic Problem-solving Errors

As a part of this diagnostic investigation, students were required to record their problem-solving algorithms on the instrument so that a 'bug' analysis could be conducted. More than one error could be recorded per algorithmic procedure. Table 1 is a percentage of errors committed, listing the identified bugs by total instrument, and then divided by calculator and no-calculator instrument types. This error analysis does not include errors caused from rounding and does not include subjects who did not show their work. The average for students not showing their work was 15.1% for questions one and three and 80% on question two. Averaging across all questions for not showing work by calculator ( $\underline{M} = 6.4$ ) and no-calculator ( $\underline{M} = 40.4$ ) illustrates that nearly half of the paper-and pencil students either did not attempt to solve the problems or did not record the solution processes. Question four is not included, in that, 86.4 % of all students could not answer this problem correctly and much of their attempted work could not be deciphered.

**Table 1***Classifications of Problem-solving Errors*

<b>Population (N = 172)</b>	<b>All</b>			<b>Calculator</b>			<b>No-calculator</b>		
	<b>Q #1</b>	<b>Q #2</b>	<b>Q #3</b>	<b>Q #1</b>	<b>Q #2</b>	<b>Q #3</b>	<b>Q #1</b>	<b>Q #2</b>	<b>Q #3</b>
Subjects that committed errors	66	25	97	30	1	32	36	24	65
Percent that did not show work	13.6%	80.0%	16.5%	6.7%	0.0%	12.5%	19.4%	83.3%	18.5%
Algorithms that were analyzed	<b>57</b>	<b>5</b>	<b>81</b>	<b>28</b>	<b>1</b>	<b>28</b>	<b>29</b>	<b>4</b>	<b>53</b>
<b>Error Type</b>									
Area Calculation	N/A	N/A	3.7%	N/A	N/A	0.0%	N/A	N/A	5.7%
Area Conversion	68.4%	N/A	4.9%	60.7%	N/A	10.7%	75.9%	N/A	1.9%
Conversion Inches to Decimal	N/A	40.0%	28.4%	N/A	0.0%	7.1%	N/A	50.0%	39.6%
Used Square Yd as Square Ft	N/A	N/A	40.7%	N/A	N/A	0.0%	N/A	N/A	62.3%
Conversion Cu Ft to Cu Yd	N/A	N/A	28.4%	N/A	N/A	32.1%	N/A	N/A	26.4%
Failed to Convert	15.8%	N/A	18.5%	28.6%	N/A	10.7%	3.4%	N/A	22.6%
Arithmetic in Division	0.0%	60.0%	9.9%	0.0%	100.0%	3.6%	0.0%	50.0%	13.2%
Arithmetic in Multiplication	8.8%	0.0%	12.3%	3.6%	0.0%	0.0%	13.8%	0.0%	18.9%
Copied from Neighbor	3.5%	0.0%	2.5%	3.6%	0.0%	7.1%	3.4%	0.0%	0.0%
Unknown	3.5%	0.0%	21.0%	3.6%	0.0%	28.6%	3.4%	0.0%	17.0%

To further illustrate the errors committed the following are image scans illustrating student errors by category see Appendix B.

### *Conclusions*

It was hypothesized that paper-and-pencil students would not be as successful as calculator students while computing their solutions. This study's results support this hypothesis. The results indicate that there were differences between the higher performing calculator group and the lower performing paper-and-pencil group. The significance of these results indicate that students cannot problem-solve in the simple unifying math concepts investigated by this study and there is an increase in problem drawing activity when students work with more difficult compound volumes.

On question one, of the 57 students who erred and showed their work the largest percentage of error resulted from the student's using an incorrect conversion factor for converting the square foot area of a surface into square yards (68.4%). They did not have the construct that there were 9 square feet in a square yard. This error was more prevalent within the no-calculator group with

75.9% committing this error. There were 15.8% that failed to convert with the calculator group most frequently committing this error (28.6%). The most common error within question two was committed by students dividing 12 or a foot by the inches of concrete thickness, with 80% of those who erred not showing their work. The errors of the first two questions compound their inability to solve the volume question. If they cannot solve for area and cannot solve for a decimal equivalency factor, they sure cannot solve for cubic yards of volume. Question three begins to exemplify the difficulty of categorizing the errors student make. Many attempted to use their square yards measure as a square foot measure (40.7%) and compounding this error was that many used a cubic yards conversion factor other than 27 cubic feet in a cubic yard (28.4%). Many failed to convert their calculations into a cubic measure at all (18.5%) by reporting the measure in square feet, cubic feet and square yards. This error was committed mostly by the no-calculator group (62.3%). The interesting differences between the two groups is the difficulty students within the paper-and-pencil group had in performing simple multiplication (18.9%) and division (13.2%) with a large portion (17.0%) falling into the “Unknown” category. Question four’s algorithms were not analyzed, in that; the amount of algorithm variance was too great and many quit most likely not attempting the solution because they are unsure of how to proceed.

Counter to the evidence presented here, students in upper level construction coursework should be able to score better than 60.5% on slab area questions, 42.4% on cubic yard volume questions, and 17.4% on volume questions compounded by ditch slope, but they do not. This generalized poor performance is not necessarily related to the type of tools used but more likely to a lack of mathematical problem solving skill. The best example of mathematical incompetence must be the student who provided 0 cu yd in response to question three: “Suppose that same parking lot in question 1 (*Answer #1 was 45,000 sf.*) had a paving thickness of 5” as in question 2 (*Answer #2 was 0.42 ft.*), how many cubic yards of paving material are in this parking lot? (*Round up to nearest whole number*). His algorithm was;  $5 \text{ in} = 1 \text{ ft} \div 12 \text{ in} = 3 \text{ yd} \div 9 \text{ ft} = 15 \div 108 = .138$ . Then .138 was rounded to 0. The correct volume answer of 695 cubic yards must have been lost in the rounding. One would be lead to believe that with eight years of concept learning in secondary education and the passing of college entrance examination that this type of mathematical problem solving would be more or less instinctive.

It is clear that within this sample population students perform significantly better on both between-subject factors when they have a calculator available. It is also clear that as the problems advance in difficulty, the students draw more images of the problem in an attempt to understand the problem through a graphic representation of its variables. This difference may be the result of cognitive loading (Sweller, 1990; Chandler & Sweller, 1991). The calculator group may have been cognitively freer to pay attention the finer points of the questions due to a reduced cognitive load resulting from the use of a calculator.

The larger question is; has the classroom computer usage caused the effects illuminated by this study? This question could be investigated by administering this instrument within an environment where calculators are not allowed and again where they are allowed. The author has and continues to collect addition data that includes modified course instruction. The current study being conducted tests the students after they have had remedial modules on these concepts. The author has also administered this instrument at the graduate level and the results are even more depressing. This might be a reflection of the fact that 95% of the graduates are from



countries that use the metric system. The instrument could use a contrived measurement system to better delineate mathematical problem solving from the lack of expertise in the US measure system.

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## **Appendix A**

### **Area, Conversion, Volume, Slope Diagnostic Instrument**

*Solve the following problems, accuracy is important. Enter your solutions and units in the boxes provided. You must use the space provided below the question to record the formula used to arrive at your solution. You have 10 minutes.*” The “No Calculator” instrument header included “*DO NOT USE a calculator*” and the “Calculator” instrument included “*USE a calculator*”.

Question 1 - Area (Square Yards) or (Length  $\times$  Width)  $\div$  9)

A-1. Suppose you have a commercial project with a parking lot measuring 100' by 300', how many square yards are in this parking lot? (*Round up to nearest whole number*)

*Paper:*  $((100. \text{ ft.} \times 300. \text{ ft.}) \div 9 \text{ sf.}) = 3,333.3 \text{ sy. or } 3,334 \text{ sy.}$

*Calculator:*  $(100. \text{ ft.} \times 300. \text{ ft.} \div 9 \text{ sf.}) = 3,333.3 \text{ sy. or } 3,334 \text{ sy.}$

B-1. Suppose you have a commercial project with a parking lot measuring 150' by 300', how many square yards are in this parking lot? (*Round up to nearest whole number*)

*Paper:*  $((150. \text{ ft.} \times 300. \text{ ft.}) \div 9 \text{ sf.}) = 5,000. \text{ sy. or } 5,000 \text{ sy.}$

*Calculator:*  $(150. \text{ ft.} \times 300. \text{ ft.} \div 9 \text{ sf.}) = 5,000. \text{ sy. or } 5,000 \text{ sy.}$

#### Question 2 - Conversion of inches into a Decimal Equivalency (Foot) or (Inches $\div$ 12)

A-2. What is the decimal equivalency of 8" in feet? (*Round up to nearest hundredths*)

*Paper & Calculator:*  $(8. \text{ in.} \div 12 \text{ in.}) = 0.667 \text{ ft. or } 0.67 \text{ ft.}$

B-2. What is the decimal equivalency of 5" in feet? (*Round up to nearest hundredths*)

*Paper & Calculator:*  $(5. \text{ in.} \div 12 \text{ in.}) = 0.417 \text{ ft. or } 0.42 \text{ ft.}$

#### Question 3 - Volume (Cubic Yards) or ((Length $\times$ Width $\times$ Depth) $\div$ 27)

A-3. Suppose that same parking lot in question 1 had a paving thickness of 8" as in question 2, how many cubic yards of paving material are in this parking lot? (*Round up to nearest whole number*)

*Paper:*  $((100. \text{ ft.} \times 300. \text{ ft.}) \times 0.667 \text{ ft.}) \div 27 \text{ cf.}) = 741.1 \text{ cy. or } 742 \text{ cy.}$

*Calculator:*  $(100. \text{ ft.} \times 300. \text{ ft.} \times (8. \text{ in.} \div 12 \text{ in.}) \div 27 \text{ cf.}) = 740.7 \text{ cy. or } 741 \text{ cy.}$

B-3. Suppose that same parking lot in question 1 had a paving thickness of 5" as in question 2, how many cubic yards of paving material are in this parking lot? (*Round up to nearest whole number*)

*Paper:*  $((150. \text{ ft.} \times 300. \text{ ft.}) \times 0.417 \text{ ft.}) \div 27 \text{ cf.}) = 695.0 \text{ cy. or } 695 \text{ cy.}$

*Calculator:*  $(150. \text{ ft.} \times 300. \text{ ft.} \times (5. \text{ in.} \div 12 \text{ in.}) \div 27 \text{ cf.}) = 694.4 \text{ cy. or } 695 \text{ cy.}$

#### Question 4 - Volume with Slope (Cubic Yards) or (((Length) $\times$ ((Depth $\div$ Rise) + (Width)) $\times$ (Depth)) $\div$ 27)

A-4. Suppose you are excavating a ditch required for the installation of a 12" water line. The square ditch depth is 5' and width is 3'. The ditch requires a side slope of a 2:1 ratio (*Rise: Run*) required for worker safety. What is the volume of soil excavated per linear foot in cubic yards? (*Round up to nearest hundredths*)

*Paper:*  $((5.5 \text{ ft.} \times 5. \text{ ft.}) \times 1. \text{ ft.}) \div 27 \text{ cf.}) = 1.019 \text{ cy. or } 1.02 \text{ cy.}$

*Calculator:*  $(5.5 \text{ ft.} \times 5. \text{ ft.} \times 1. \text{ ft.} \div 27 \text{ cf.}) = 1.019 \text{ cy. or } 1.02 \text{ cy.}$

B-4. Suppose you are excavating a ditch required for the installation of a 12" water line. The square ditch depth is 6' and width is 4'. The ditch requires a side slope of a 2:1 ratio (*Rise: Run*) required for worker safety. What is the volume of soil excavated per linear foot in cubic yards? (*Round up to nearest hundredths*)

Paper:  $((7. \text{ ft.} \times 6. \text{ ft.}) \times 1. \text{ ft.}) \div 27 \text{ cf.} = 1.556 \text{ cy. or } 1.56 \text{ cy.}$   
 Calculator:  $(7. \text{ ft.} \times 6. \text{ ft.} \times 1. \text{ ft.} \div 27 \text{ cf.}) = 1.556 \text{ cy. or } 1.56 \text{ cy.}$

### Appendix B Algorithmic Error Examples

$333 \text{ yd}^2$

$$L \times W = SA \quad 110 \times 300 = 33000$$

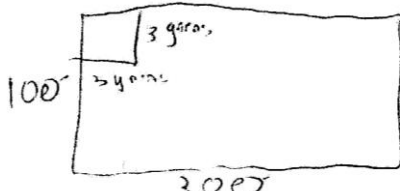
$$3,330^2 \text{ yd}^2 \quad 3' = .33 \text{ yd}^2$$

$212 \text{ yd}^2$

$$\sqrt{(150)(300)} = 212.132$$

Figure 4: Area Calculation (100 ft. or 150 ft.  $\times$  300 ft.)

$31 \text{ square yards}$



$L \times W = 100' \times 300' = 30,000 \text{ sq ft}$   
 $30,000 \text{ sq ft} \div 9 \text{ sq ft} = 3,333 \text{ sq yd}$   
 $\sqrt{30,000} = 31$

Figure 5: Area Conversion Value Square Feet to Square Yard – (9 sf. per 1 sy.)

$2.1$

$$\sqrt[5]{12.0} = \frac{10}{2.0}$$

$0.134$

$$36 \overline{) 5.000} \quad \begin{array}{r} .138 \\ 36 \\ \underline{140} \\ 108 \\ \underline{0320} \end{array} \quad \begin{array}{r} 36 \\ \underline{180} \\ 144 \\ \underline{108} \\ 36 \\ \underline{324} \end{array}$$

Figure 6: Conversion Inches to Decimal Measure – (Inches / 12)

$7,771 \text{ yd}^3$

$$3 \overline{) 23,337} \quad \begin{array}{r} 23,337 \\ \underline{24} \\ 40 \\ \underline{36} \\ 40 \end{array} \quad \begin{array}{r} 36000 \\ \underline{8} \\ 288,000 \end{array} \quad \begin{array}{r} 3000 \\ \underline{1} \\ 30000 \end{array}$$

Figure 7: Used Square Yards as Square Feet

$$2,200 \text{ yd}^3$$

$$\frac{.67 \text{ ft}}{3 \text{ ft}} = .22 \text{ yd}$$

$$10,000 \text{ yd}^2 \times .22 \text{ yd} = 2,200 \text{ yd}^3$$

Figure 8: Volume Conversion Value Cubic Feet to Cubic Yard – (27 cf. per 1 cy.)

$$30,000 \text{ yd}^2$$

$$100 \times 300 = 30,000$$

$$18,900 \text{ yd}^3$$

$$45,000 \times .42 = 18,900 \text{ cubic yds}$$

Figure 9: Failed to Convert

$$3,313$$

$$300 \times 100 = 30,000 / 9$$

$$\begin{array}{r} 3,313 \\ 27 \overline{) 30,000} \\ \underline{27} \phantom{000} \\ 30 \phantom{00} \\ \underline{27} \phantom{00} \\ 300 \\ \underline{270} \\ 300 \\ \underline{270} \\ 300 \end{array}$$

$$\begin{array}{r} 1000 \\ 3000 \\ \hline 10000 \\ 30000 \\ \hline 100000 \\ 300000 \\ \hline 1000000 \\ 3000000 \\ \hline 10000000 \\ 30000000 \end{array}$$

Figure 10: Arithmetic Procedure Division

$$\text{[Empty Box]}$$

$$\begin{array}{r} 30,000 \\ \times .667 \\ \hline 210000 \\ 2400000 \\ \hline 24000000 \\ \hline 26610000 \end{array}$$

$$\begin{array}{r} 8 \quad 2 \\ 12 \quad 3 \\ \hline 27 \overline{) 2661000} \end{array}$$

$$9900 \text{ CF}$$

$$330 \text{ CY}$$

$$\begin{array}{r} 27 \overline{) 9900} \\ \underline{27} \phantom{00} \\ 22 \phantom{00} \\ \underline{27} \phantom{00} \\ 50 \phantom{00} \\ \underline{54} \phantom{00} \\ 40 \phantom{00} \\ \underline{45} \phantom{00} \\ 169 \end{array}$$

$$\begin{array}{r} 30000 \times .667 \\ \hline 90000 \\ \hline 1980000 \\ \hline 9900.00 \end{array}$$

Figure 11: Arithmetic Procedure Multiplication

$$3,333 \text{ yd}$$

$$150 \times 300 = 45,000 \text{ SA}$$

Figure 12: Copied from Neighbor (100 ft. x 300 ft. instrument)

$$620,000 \text{ yd}$$

$$\begin{array}{r} 10000 \\ \times .62 \\ \hline 20000 \\ 60000 \\ \hline 620000 \end{array}$$

$$3 \text{ cubic yds}$$

$$\begin{array}{r} 10,000 \\ \hline 80,000 \quad 27 \overline{) 80,000} \end{array}$$

$$\begin{array}{r}
 \boxed{210,000 \text{ cu. yds.}} \\
 + 15,000 \\
 \hline
 135,000 \\
 + .42 \\
 \hline
 270,000 \\
 + 540,000 \\
 \hline
 810,000
 \end{array}
 \qquad
 \begin{array}{r}
 210,000 \\
 27 \overline{) 567,000} \\
 \underline{54} \\
 27 \\
 \underline{27} \\
 0
 \end{array}$$

Figure 13: Unknown, Total Loss of Perspective Error

$$\begin{array}{r}
 \boxed{104,20,100 \text{ cu. yds.}} \\
 30,000 \text{ SF } (.67 \text{ F}) \\
 \hline
 27,000 \\
 180,000 \\
 \hline
 107,000
 \end{array}
 \qquad
 \begin{array}{r}
 30,000 \\
 .67 \\
 \hline
 20,100
 \end{array}$$

Figure 14: Unknown, What the Hey, That Can't Be Right Error